

# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

FEB 21 1947

TECHNICAL NOTE

No. 1154

A COMPARISON OF THE LATERAL MOTION CALCULATED  
FOR TAILLESS AND CONVENTIONAL AIRPLANES

By Charles W. Harper and Arthur L. Jones

Ames Aeronautical Laboratory  
Moffett Field, Calif.



Washington  
February 1947

OFFICE OF THE SECRETARY  
LANGLEY MEMORIAL AERONAUTICAL  
LABORATORY  
Hampton Field, Va.



# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE NO. 1154

### A COMPARISON OF THE LATERAL MOTIONS CALCULATED FOR TAILLESS AND CONVENTIONAL AIRPLANES

By Charles W. Harper and Arthur L. Jones

#### SUMMARY

A theoretical analysis of the lateral dynamic motion of tailless and conventional airplanes was made for airplanes of two classes - fighter and heavy transport. Their reactions to a lateral gust and the control power required (in coefficient form) by each for simple maneuvers were determined and compared.

It is shown that no great difference should be expected between the lateral motions that characterize the stability of the two types of airplanes. The tailless airplanes show the greatest displacements for a given disturbance and have the least damping in the oscillatory mode. It appears unlikely that these oscillations can be made as small or as highly damped as for a conventional airplane. It is estimated that some difficulty will be met in satisfying the requirement that lateral oscillations with controls free damp to one-half amplitude in 2 cycles.

The two types of airplanes require almost identical aileron control power to perform a given maneuver. However, the tailless airplane required only about one-half to one-third of the directional control power of the conventional airplane to perform a given maneuver. While this is an advantage insofar as directional control requirement is concerned, the low damping in yaw which is largely responsible for this effect makes the airplane extremely susceptible to yawing disturbances that are normally considered unimportant.

#### INTRODUCTION

Acceptable dynamic characteristics of conventional airplanes are obtained partially through certain design criterions which have been established, and partially through a trial-and-error process based on past experience. This latter process has been extensively employed to

choose the lateral characteristics of airplanes because of a lack of precise flying qualities requirements. In the case of the tailless airplane, however, little experience exists to guide the choice of lateral characteristics. This fact was emphasized during tests of a model of a tailless airplane when it became apparent that only the vaguest evaluation could be made of the suitability of various measured aerodynamic characteristics.

It was thought that a theoretical investigation of the lateral-control power required and the lateral-dynamic-stability characteristics of a conventional and tailless airplane might clarify the situation to some extent. A comparison of the control power required was obtained by subjecting the two types of airplanes to certain predetermined maneuvers through the medium of the dynamic-motion calculations outlined in this report under Method. These maneuvers and a gust condition applied to these airplanes to reveal their inherent dynamic-stability characteristics are described under Procedure.

Sufficient wind-tunnel data were at hand to make these calculations for the tailless airplane and for a conventional airplane of comparable size and expected performance. Wind-tunnel data were also available for tailless and conventional airplanes of the heavy transport class. A similar analysis was made for these airplanes. It should be emphasized that the results of these analyses apply to airplanes more or less typical of their class, and the results should not be considered as an absolute measure of the relative behavior of tailless and conventional designs.

The stability characteristics in the form of stability derivatives were either measured or calculated for the individual airplanes. With one exception no attempt was made to evaluate the effect of changing these characteristics, since it was thought that they were typical. The one exception found necessary to consider was the variation in the yawing moment which is due to yawing of the tailless airplanes. It can be shown that, where vertical fins are mounted on the wings, the yawing moment, which is due to yawing, varies appreciably with the angle of yaw; whereas for a conventional airplane this effect is quite small. Thus it might be expected that the motion of a tailless airplane will vary between the two extremes of the three cases presented, rather than closely following any one.

#### PROCEDURE

Four types of lateral motion were considered: (1) that following entrance into a sharp-edged lateral gust with controls fixed, (2) zero sideslip turns, (3) a  $5^\circ$  change in heading made with the rudder alone,

and (4) a roll to approximately 90° bank and return, using ailerons alone. The first two motions were investigated for all four airplanes and the last two for the small airplanes only. It was not expected that these maneuvers would represent flight conditions exactly, but it was believed that they would show where appreciable differences existed between the dynamic lateral control and -stability characteristics of tailless and conventional airplanes. In each case the airplane was considered to be near or at high speed. The altitudes assumed were sea level for the large airplanes and 22,000 feet for the fighters.

The inherent dynamic characteristics of an airplane are shown most clearly by its reaction to an outside disturbance with the controls fixed. A sharp-edged lateral gust was considered the most likely form of outside disturbance that would be encountered in flight. Consequently, for each airplane the complete lateral motion was computed for a 10-second period following its entrance into a sharp-edged lateral gust.

It was thought that any differences in the control requirements for each type of airplane could be seen through the difference in control coefficients required to make a zero sideslip turn. These coefficients, therefore, were computed for each airplane for turns extending over varying intervals of time but having equal maximum angles of bank (30°).

The two additional investigations were made on small airplanes because it was believed that these would show to some extent the ease with which each type could be rapidly maneuvered. Flight experience has shown that fighter airplanes unable to make a slight change in heading without extensive control coordination are unsatisfactory. It was expected that the relative amounts of coordination required for the two types of airplanes could therefore be judged from the relative amounts of sideslip and roll developed in each case. Similar reasoning led to the investigation of the rapid roll maneuver.

#### COEFFICIENTS AND SYMBOLS

The coefficients and symbols defined herein are referred to the system of stability axes in which the X-axis is in the plane of symmetry and is parallel to the relative air stream, the Z-axis is in the plane of symmetry and is perpendicular to the X-axis, and the Y-axis is perpendicular to the plane of symmetry. The coefficients and symbols are defined as follows:

$C_L$  airplane lift coefficient  $\left( \frac{\text{lift}}{qS} \right)$

$C_l$  rolling-moment coefficient  $\left( \frac{\text{rolling moment}}{qSb} \right)$

$C_n$	yawing-moment coefficient $\left( \frac{\text{yawing moment}}{qSb} \right)$
$C_y$	side-force coefficient $\left( \frac{\text{side force}}{qS} \right)$
$L$	rolling acceleration $\left( \frac{\text{rolling moment}}{I_{xx}} \right)$
$N$	yawing acceleration $\left( \frac{\text{yawing moment}}{I_{zz}} \right)$
$Y$	side-force coefficient $\left( \frac{\text{side force}}{m} \right)$
$m$	mass of airplane, slugs
$I_{xx}$	moment of inertia about X-axis, slug-feet square
$I_{zz}$	moment of inertia about Z-axis, slug-feet square
$\rho$	air density, slugs per cubic foot
$b$	wing span, feet
$S$	wing area, square feet
$l$	distance from the center of gravity of the airplane to the hinge line of the vertical tail, feet
$\alpha$	effective airplane angle of attack, radians
$g$	acceleration due to gravity, feet per second squared
$\lambda_n$	the $n^{\text{th}}$ root of the stability quartic
$U_0$	velocity along X-axis, feet per second
$q$	dynamic pressure $\left( \frac{1}{2} \rho U_0^2 \right)$ , pounds per square foot
$v$	sideslipping component of velocity, feet per second
$p$	rolling velocity, radians per second
$r$	yawing velocity, radians per second
$\phi$	angle of bank, radians except as otherwise indicated

- $\beta$  angle of sideslip, radians except as otherwise indicated
- $\psi$  angle of yaw, radians except as otherwise indicated
- $C_{l_\beta}$  rate of change of rolling-moment coefficient with angle of sideslip  $(\partial C_l / \partial \beta)$ , per degree
- $C_{n_\beta}$  rate of change of yawing-moment coefficient with angle of sideslip  $(\partial C_n / \partial \beta)$ , per degree
- $C_{Y_\beta}$  rate of change of side force coefficient with angle of sideslip  $(\partial C_Y / \partial \beta)$ , per degree
- $C_{l_p}$  rate of change of rolling-moment coefficient with wing-tip helix angle  $(\partial C_l / \partial \frac{pb}{2U_0})$
- $C_{n_p}$  rate of change of yawing-moment coefficient with wing-tip helix angle  $(\partial C_n / \partial \frac{pb}{2U_0})$
- $C_{l_r}$  rate of change of rolling-moment coefficient with  $rb/2U_0$   $(\partial C_l / \partial \frac{rb}{2U_0})$
- $C_{n_r}$  rate of change of yawing-moment coefficient with  $rb/2U_0$   $(\partial C_n / \partial \frac{rb}{2U_0})$
- $L_\beta$  rate of change of rolling acceleration with angle of sideslip  $(C_{l_\beta} \frac{180}{\pi} \frac{qSb}{I_{xx}})$
- $N_\beta$  rate of change of yawing acceleration with angle of sideslip  $(C_{n_\beta} \frac{180}{\pi} \frac{qSb}{I_{zz}})$
- $Y_\beta$  rate of change of side-force acceleration with angle of sideslip  $(C_{Y_\beta} \frac{180}{\pi} \frac{qS}{m})$
- $L_p$  rate of change of rolling acceleration with rate of roll  $(C_{l_p} \frac{b}{2U_0} \frac{qSb}{I_{xx}})$

$L_r$  rate of change of rolling acceleration with rate of yaw

$$\left( C_{l_r} \frac{b}{2U_0} \frac{qSb}{I_{xx}} \right)$$

$N_p$  rate of change of yawing acceleration with rate of roll

$$\left( C_{n_p} \frac{b}{2U_0} \frac{qSb}{I_{zz}} \right)$$

$N_r$  rate of change of yawing acceleration with rate of yaw

$$\left( C_{n_r} \frac{b}{2U_0} \frac{qSb}{I_{zz}} \right)$$

#### METHOD

This section is intended as a guide and reference to the methods used and the data needed in making the dynamic calculations required for the procedures just discussed. Since this type of calculation is quite laborious and complicated and is not generally familiar, the details of these calculations will not be of great interest or concern to anyone who has not made or does not contemplate making such calculations.

The stability derivatives for each airplane were obtained from wind-tunnel tests made in the Ames 7- by 10-foot wind tunnel and from reference 1. For the two tailless airplanes the three values of  $C_{n_r}$ , which were due to the vertical tail, were estimated from the results of oscillation experiments made on a model of a tailless airplane. The physical properties of the airplane were obtained from the manufacturer. These data are presented in table I. The relative sizes and forms of the airplanes investigated are shown in figure 1.

The method used to compute the motion of the airplane following a unit acceleration is outlined in the appendix of this report. The expressions for the constants required in the equations of motion are tabulated in the appendix and their values for the individual airplanes are presented in table II.

The method for compounding the effects of unit disturbances to obtain the motion resulting from a gust is presented in reference 2. The approximations made therein apply equally to this report. For the conventional airplanes, account was taken of the period of penetration of the airplane into the gust to account for the delay of the reactions due to the tail. No delay of any type was considered for

the tailless airplanes. In addition, the motion of the airplane with respect to the ground was determined as outlined in the appendix.

In order to find the yawing acceleration required to make and hold a  $5^\circ$  change in heading, the method outlined in reference 3 was followed. It is possible to determine the motion resulting from an irregular disturbance by using a graphical solution of Carson's integral when the motion resulting from unit disturbance is known. In a similar manner it is possible to determine the necessary variation of the irregular disturbance when the desired motion and the motion resulting from unit disturbance are known. The necessary variation of yawing acceleration with time was first determined such that the airplane experienced a  $5^\circ$  change in heading in  $1\frac{1}{2}$  seconds and maintained this heading thereafter. The oscillatory tendencies of the airplane were thus reflected in the required yawing acceleration which oscillated rapidly about a mean value. It was considered unlikely that a pilot would perform a corresponding control maneuver, and hence the yawing acceleration was varied approximately as the mean of the oscillatory curve previously determined. The airplane was then free to oscillate, the only restrictions being that a  $5^\circ$  change in heading was reached in  $1\frac{1}{2}$  seconds and that at no time during an oscillation did the airplane deviate more than  $0.5^\circ$  from this heading.

The roll maneuver and the required rolling accelerations of the airplane were determined in a manner similar to that previously discussed. The approximation made in the equations of equilibrium that  $\sin \phi = \phi$  introduces an appreciable error into these results where  $\phi$  becomes as great as  $90^\circ$ . Accordingly, the displacements computed for this maneuver should not be expected to predict closely those that would be measured in flight. It is believed, however, that this error does not invalidate the conclusions drawn as to the difference between the motions of tailless and conventional airplanes.

It is not necessary to solve the equations of motion for a unit disturbance to determine the rolling and yawing accelerations required to perform a perfect (zero sideslip) turn. The procedure followed was that outlined in references 4 and 5 wherein the variation of the angle of bank with time is predetermined, zero sideslip specified, and the necessary yawing and rolling accelerations computed from the equation of equilibrium. For each airplane the maximum angle of bank was held at  $30^\circ$  and the length of time in which the maneuver was completed was varied. The yawing and rolling accelerations required to perform the maneuver were then reduced to standard NACA moment-coefficient form.

All motions are represented in terms of displacement (angles or distance) rather than rates of displacement (velocities) for easier visualization.

## DISCUSSION

The complete dynamic lateral motion of an airplane can be expressed analytically as three modes of the motion which are functions of the roots of the stability equation.

It is from these roots that the damping and, in one case, the period of these three modes of lateral motion are obtained. A discussion of these roots and of their significance is presented in this section along with the discussions of the results of the investigations of the maneuvers and gust conditions to which the airplanes were subjected by mathematical simulation.

A discussion of the motions resulting from unit accelerations applied to the airplane is also included in this section. These motions are the basic variations of the motion of the airplane resulting from an external disturbance and can be compounded into the motion resulting from any explicit gust or control-surface deflection.

### Roots of the Stability Equation and Their Significance

For the degrees of freedom considered herein, the stability equation is a quintic having, generally, two real roots, a pair of conjugate complex roots, and a zero root (indicating no inherent tendency of the airplane to hold a particular compass heading). One of the real roots ( $\lambda_1$ ) is small, corresponds to a slow turning and banking motion, and can be negative (spiral stability) or positive (spiral instability). The other real root ( $\lambda_2$ ) is, at low lift coefficients, large and negative and corresponds to a highly damped rolling motion of the wings relative to the air. The pair of conjugate complex roots correspond to a combined rolling, yawing, and sideslipping oscillation ("Dutch roll"). The real part of this root is usually small and can be negative (oscillatory convergence) or positive (oscillatory divergence).

The values of the various roots, the time to increase or decrease the amplitude a given amount, and the period of the oscillations are given for each airplane in table III. Airplanes 1, 2 (cases a and b), and 4 (case a) show spiral instability. In no case is this sufficiently pronounced to be considered objectionable. It should be noted that the spiral instability of the tailless airplanes occurs because of a low damping in yaw ( $C_{nr}$ ) rather than high directional stability as in the usual case. The resistance to rolling is high for each case considered and hence this motion is so

rapidly damped that it may be ignored after a fraction of a second. The extremely high damping exhibited by airplane 3 is due to a low moment of inertia in rolling and not to differing aerodynamic characteristics.

The most significant difference between the motions of the conventional and tailless airplanes appears in the oscillatory motion. This motion is damped for all the airplanes but is much more rapidly damped for the conventional airplanes.

The period of oscillation of the tailless airplanes is about 50 percent greater than that of the conventional airplanes. It was found that increasing the damping ( $C_{nr}$ ) of the tailless airplane has little effect on the period of the oscillations but appreciably increases the damping of this motion. A requirement for satisfactory flying qualities states that the control-free lateral oscillation should always damp to one-half amplitude in 2 cycles. (See reference 6.) With controls fixed, airplane 1 damps to one-half amplitude in one-half cycle and airplane 3 in 1 cycle. It is probable that with controls free these airplanes would satisfy the preceding requirement. Airplane 2 requires 2 cycles to damp to one-half amplitude with the lowest value of  $C_{nr}$  and one-half cycle with the highest values of  $C_{nr}$ . The reduction in directional stability with controls free would therefore make the airplane unsatisfactory in this respect with the lowest and probably with the medium values of  $C_{nr}$ . Airplane 4 requires 4 cycles to damp to one-half amplitude with the lowest value of  $C_{nr}$  and  $1\frac{1}{2}$  cycles with the highest value of  $C_{nr}$ . It is doubtful that this airplane would meet the preceding requirements with the controls free.

#### Motion Resulting from Unit Disturbances

The variations with time of the motions (in roll, yaw, and sideslip) resulting from each of the three possible unit lateral accelerations are shown in figures 2 to 19. These motions are used in compounding the other motions investigated in the dynamic-motion calculations. Consequently, they are individually significant and reveal directly interesting characteristics of the stability of the airplane. The unit accelerations for the various airplanes correspond to the following moment and force coefficients:

Airplane	$C_l$	$C_n$	$C_y$
1	0.0541	0.0870	0.0186
2	.0332	.0433	.0187
3	.0018	.0057	.0041
4	.0038	.0041	.0032

The variations in the size of the coefficients noted in the preceding table are due to the variations in mass and moments of inertia of the airplanes. It is evident, therefore, that a direct comparison of the magnitudes of the displacements for a given unit acceleration is not valid. However, since the magnitudes of the displacements vary directly with the magnitudes of the disturbances, it is a simple matter to estimate the various displacements at a constant coefficient for purposes of comparison. It should be noted that the displacement scales used in figures 2 to 19 are not consistent. Each scale was chosen such that the variation of the motion could easily be seen and compared between airplanes. The relative magnitude of the displacements resulting from unit accelerations in roll, yaw, and sideslip are shown in figures 20 to 25 for the conventional airplanes but are typical also of the tailless airplanes.

A discussion of this motion falls logically into two parts, the transitory and the eventual motions. The first is governed almost entirely by the oscillatory mode and slightly by the heavily damped rolling represented by the  $\lambda_2$  root. Provided oscillatory convergence exists, the eventual motion is governed entirely by the spiral stability or instability (represented by the  $\lambda_1$  root).

The longer oscillatory period and lower oscillatory damping of the tailless airplanes are quite evident from these figures. These characteristics are most noticeable in the curves showing the motion resulting from a side acceleration and in sideslipping, but the relative magnitude of the various motions must be considered. Where these oscillations are large compared to the over-all motion, it is apparent that the increase in  $C_{nr}$  of the tailless airplane has a major effect. Where the oscillations are small compared to the over-all motion, the value of the damping, which is due to yawing, has a negligible effect.

For both types of airplanes, positive displacements result from positive acceleration with the exception of  $\phi_Y$  and  $\beta_R$ , in which cases negative displacements result from positive accelerations. In the case of  $\psi_L$ , positive accelerations cause a momentary negative displacement, after which the displacement becomes and remains positive. This initial negative displacement is due to the negative yawing moment resulting from a positive rolling velocity.

The eventual motion of the various airplanes is wholly independent of type, depending only on the existence or nonexistence of spiral stability. With a steady application of positive rolling or yawing unit accelerations, the spirally stable airplanes reach a constant positive angle of bank and sideslip and a constant rate of yawing. A positive unit lateral-force acceleration steadily applied to a spirally stable airplane will eventually produce a constant negative

angle of bank and a constant positive angle of yaw but no sideslip. The steady application of any of the positive unit accelerations to the spirally unstable airplanes results in constantly increasing positive angles of bank, yaw, and sideslip. Little value can be gained through a study of the various magnitudes of these angular displacements since they rapidly exceed the limits imposed by the basic assumptions in the theory of dynamic calculations.

#### Motion Resulting from a Sharp-Edged Lateral Gust

The time histories of the angles of bank, yaw, and sideslip about axes fixed in the airplanes are presented in figures 26 to 31 for a side-gust velocity of 10 feet per second. The amplitudes of the angular deflections are directly proportional to the size of the applied disturbance. Consequently, these results would have the same characteristics but different magnitudes for other gust velocities.

Transitory motion.- In general, the mean angular displacements for the small airplanes are very small. The oscillatory motions though predominant and rapid are convergent. For the conventional airplane the rate of damping is greater than for any case of the tailless.

A comparison of the large tailless and conventional airplanes discloses the same differences in lateral-stability characteristics as previously stated. Their periods of oscillation are from three to four times greater than for the small airplanes, but their rates of damping are slightly less.

For all airplanes the initial angle of sideslip upon entering the gust is  $v/U_0$  positive when the gust is from the right. Consequently, the angle of zero sideslip of the airplane in the gust is at  $-v/U_0$  as shown on the  $\beta$  scale of figures 28 and 31. The tailless and the conventional airplanes oscillate in yaw about a heading different from their original heading by an amount equal to approximately the initial angle of sideslip of the airplane. All oscillations in bank have a mean value very close or equal to zero.

Eventual motion.- For a gust, as for unit disturbances, the eventual motion of the airplane investigated is largely a function of the spiral stability. The stable cases will gradually recover to zero angles of bank and sideslip (with respect to the gust) and will eventually return to their original headings. This recovery in heading takes an infinite length of time, and this theoretical tendency of the completely stable airplane exists no matter how many different disturbances the airplane encounters.

The spirally unstable airplanes considered all turn to the right (into the wind) with the exception of the large conventional airplane which turns down-wind. This down-wind turn results from the delay of the vertical-fin yawing reaction which is due to the time required for the tail of the airplane to penetrate the gust. The direction in which an unstable airplane turns can be shown to be a function of this time delay and hence a function of airplane size and forward velocity.

Motion with respect to the ground.— The flight paths and angular displacements of the airplanes with respect to a set of axes fixed in the earth are shown in figures 32 and 33 for the first 10 seconds following entrance into a 30-foot-per-second lateral gust. Since the ratios of forward speed to gust velocity considered are large, the displacements vary almost directly with gust velocity and the higher gust velocity can be used to amplify the airplane motion.

These sketches of the flight paths give a visual picture of the resulting motion and bring out two points: (1) Spiral instability to the degree evident in these airplanes is not enough to cause any serious difficulty in restoring an airplane to its undisturbed orientation when it encounters a gust, and (2) For both sizes considered, the linear-side displacements of the tailless airplanes are the smallest due to their greater oscillations in yaw and sideslip.

#### Coordinated Turns

A typical variation of the angle of bank and the resulting rates of rolling and yawing for a zero sideslip turn are shown in figure 34. The control accelerations in roll and yaw needed to perform this maneuver have been converted to moment coefficients and are presented in figures 35 and 36. The following table gives the approximate changes in headings of the large and small airplanes for the various periods of time used in performing this maneuver.

Airplane	Period (sec)	Change in heading (deg)
1 and 2	60	124.5
1 and 2	30	62.3
1 and 2	15	31.1
3 and 4	15	13.1
3 and 4	7	6.1
3 and 4	2	1.8

The maximum accelerations required in both rolling and yawing become larger as the periods grow smaller. For both size and a given period, the relative magnitudes of the rolling-moment coefficients required are nearly equal for the two types of airplanes. This effect is reasonable since aerodynamic characteristics in roll are not greatly affected by a tail.

The yawing-moment coefficients for the small airplanes are negligible and have not been presented. For the large airplanes the yawing-moment coefficients presented in figure 36 are also quite small. The large conventional airplane needs from two to three times the maximum yawing-moment coefficients that the tailless needs with either the maximum or minimum damping in yaw considered. Based on equal control effectiveness, the large conventional airplane would therefore require slightly greater rudder and aileron coordination than the large tailless airplane. The difference in coordination required of the small airplanes can be considered negligible.

It should be noted that the term "coordination" as used in the previous paragraph means the relative amounts of aileron and rudder deflection required of the pilot to make zero sideslip turns. No consideration has been given to the ability of the pilot to make the required control motions speedily and precisely. It is entirely possible that if this factor were considered a measure of the coordination required, the preceding conclusions would be radically altered. For instance, the coordination required by the small airplane, when interpreted as the control deflection magnitude, is negligible as previously stated. When interpreted as the physical coordination required of the pilot to perform precisely these control manipulations, the coordination required may be considered large or perhaps impossible. Further studies are certainly needed to establish some norm of physical coordination before final conclusions can be drawn as to the true handling qualities of these and other airplanes.

Since the rolling-moment coefficients required of the two types of airplanes were so similar, the effects of yaw which are due to the ailerons were not investigated. It should be noted, however, that the yawing control required of the tailless airplanes is so low that an appreciable amount of favorable yaw which is due to the ailerons will require rudder deflections opposing the turn.

The small amount of directional control required of the tailless airplanes points to a possible unsatisfactory characteristic of this type of airplane. Given a rudder having conventional effectiveness it would be easy for a pilot to inadvertently overcontrol and thus initiate and possibly enforce the large, slightly damped oscillation shown in the motion resulting from a unit disturbance.

### Rudder Turn

The results of the rudder-turn calculations for the small airplanes are presented in figure 37 showing the yawing acceleration variation required for this maneuver and the resulting angles of yaw, bank, and sideslip. As previously stated, it was desired to make a  $5^\circ$  change of heading in  $1\frac{1}{2}$  seconds and to maintain that heading within half a degree thereafter by a smooth variation of the applied yawing acceleration. In two of the three cases investigated these specifications were met, but for the case of the tailless airplane with the least damping in yaw ( $C_{nr} = -0.005$ ) no smooth yawing acceleration could be found that maintained the change in heading within half a degree. All the applied yawing accelerations vary in a similar manner; a positive loop for the first 4 seconds and a negative loop that has not recrossed the axes by 10 seconds. These accelerations would eventually become positive again before dying out.

For the first 4 seconds of this maneuver the conventional airplane required approximately 2.5 times the yawing-moment coefficient, developed about the same angle of sideslip and banked slightly further than the tailless airplane. These 4 seconds would give a fighter, traveling 100 miles per hour faster than its target,  $2\frac{1}{2}$  seconds firing time while closing in from 200 to 75 yards. To maintain this change in heading the conventional airplane requires a more rapid but considerably less sustained control motion. The resulting oscillatory motion as well as the displacements in bank and sideslip vanish much sooner than they do for the tailless airplane.

### Aileron Roll

The results of the calculations made to determine the necessary variations in rolling acceleration and the resultant angles of yaw and sideslip during a roll to  $90^\circ$  bank and return to  $0^\circ$  are presented in figure 38. The effects of yaw which are due to the ailerons have been ignored in these results. It should be remembered that these displacements exceed the limitations imposed by the theory and hence only the relative magnitudes should be considered. Since the results are largely qualitative, the only case investigated for the tailless airplane was that of the highest value of damping which is due to yawing.

The variations with time of the control rolling-moment coefficients required to complete the maneuver are nearly identical for the two airplanes. They are of the same form as the corresponding curves computed for the coordinated turn maneuver and involve first a positive and then a negative control motion of equal magnitude.

The resultant oscillatory motions in yawing and sideslipping are small for both airplanes, but the tailless again shows slightly longer and less damped oscillations. The yawing motion is delayed until the maximum angle of bank is reached after which it becomes positive and opposes the return to zero bank. The sideslipping motion is positive throughout the major portion of the rolling motion, thus opposing the initial rolling and aiding in the return.

Since it is unreasonable not to expect yaw due to ailerons, the effects of both adverse and favorable yaw due to ailerons were investigated. Yawing acceleration curves based on yawing-moment coefficients equal to 10 percent of the rolling-moment coefficients are shown for favorable yaw in figure 39 together with the banking, yawing, and sideslipping displacements they effected. While the rolling-moment coefficients and hence the resultant yawing-moment coefficients were almost equal for the two airplanes, the yawing acceleration experienced by the tailless airplane was greatest due to its smaller moment of inertia. This effect, together with the lower damping in yaw, resulted in the tailless airplane exhibiting large and prolonged oscillations. While these oscillations also appeared in the motion of the conventional airplane when yaw, which was due to the ailerons, was considered, they were much less pronounced and more highly damped.

The relative magnitudes of the combined motions resulting from roll and aileron yaw may be seen by comparing figures 40 and 41 which show, for the tailless and conventional airplanes, the angles of sideslip, yaw, and roll developed. It appears from these results that tailless design will require a careful consideration of the yaw, which is due to the ailerons, if satisfactory flying qualities are to be obtained. It is probable that the beneficial effects usually associated with favorable yaw, which is due to the ailerons, may be for a tailless design more than counteracted by the undesirable resultant oscillations.

The curves show that the yawing and sideslipping motions are respectively positive and negative when favorable yaw is present and that the maximum angle of bank (also the corresponding rolling velocity) is increased by these beneficial effects. Adverse yaw which is due to the ailerons produces, of course, exactly the opposite results.

#### CONCLUDING REMARKS

The dynamic lateral motion of the tailless airplanes considered does not differ greatly from that of the conventional airplanes of comparable size. Both types showed oscillatory convergence and, for the various cases, possessed spiral divergence or convergence in about the same degree. The tailless airplanes showed the least damping and greatest

displacements in the oscillatory form of the motion due to any external disturbance. It appears doubtful that this motion can be made as slight for a tailless airplane as for a conventional airplane. It was estimated that the criterion for satisfactory damping of lateral oscillations with controls free (that the amplitude be damped one-half in 2 cycles) would be satisfied by both conventional airplanes, and by the large tailless airplane with the high and possibly with the medium values of  $C_{nr}$  and the small tailless airplane with the highest value of  $C_{nr}$ . The small tailless airplane probably would not satisfy this criterion when  $C_{nr}$  was reduced.

The reaction of the tailless airplanes to an outside disturbance such as a gust would not be appreciably different from the reaction of a conventional airplane. Therefore, no unusual lateral-control requirements should be encountered in normal steady flight.

Investigation of the control coefficients required to complete a zero sideslip turn showed that little difference would exist between the rolling coefficients required by a tailless airplane and by a conventional airplane. Two to three times less directional control was required for a tailless airplane and it will, therefore, show to a greater extent the effects of adverse or favorable yaw of the ailerons.

The tailless airplane will require considerably less directional control and will have slightly different motion where a change of heading is made with rudder alone. During the initial period of such a maneuver, the tailless airplane will have a definite advantage since the required directional control is one-half to two-thirds that of the conventional airplane. To maintain the change in heading, however, the tailless airplane will require a considerably longer period of control manipulation than will the conventional airplane.

When the yaw which is due to the ailerons is neglected, the lateral control required and the resulting sideslip and yawing developed during a rapid rolling maneuver made with ailerons alone will be nearly identical for the two types of airplanes. Aileron yaw, however, undesirably amplifies the oscillations of the tailless airplane; whereas it has only a negligible effect on the conventional airplane.

In general, it can be expected that the mean value of the lateral displacements in disturbed motion will be of the same magnitude for a tailless and a conventional airplane, but the oscillatory motion of the tailless airplane will be greater and considerably less damped. For a desired degree of maneuverability, the tailless airplane will require from one-half to one-third less directional control than the conventional airplane but will be much more significantly affected by small yawing

moments such as would result from aileron yaw, asymmetry of power, construction, and so forth.

Ames Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Moffett Field, Calif., August 5, 1946.

#### APPENDIX

An investigation of the lateral-dynamic motion of an airplane can be made by following a mathematical procedure presented in reference 4 and modified in reference 7. This method employs unit disturbances, as developed in Heaviside's operational calculus. The effects of the unit disturbances can be compounded to give motions resulting from any form of outside disturbance. The unit disturbances considered in lateral motion are unit accelerations about the X-axis (rolling) and the Z-axis (yawing) and along the Y-axis (sideslipping).

Each disturbance enforces rolling, yawing, and sideslipping motions. Thus, to define completely the motion resulting from an arbitrary disturbance, nine equations are required, identical in form but with varying constants. For example, the equation for the rolling velocity  $p$  resulting from a unit yawing acceleration  $N$  may be expressed as

$$p_N(t) = p_{N0} + p_{N1} e^{\lambda_1 t} + p_{N2} e^{\lambda_2 t} + p_{N3} e^{\lambda_3 t} + p_{N4} e^{\lambda_4 t}$$

where  $\lambda$  is a root of the stability equation and  $p_{N0}$ ,  $p_{N1}$ , and so forth, are constants involving the roots and airplane characteristics.

Heaviside's expansion theorem forms the general expression for the nine equations and from it the expressions for the constants,  $p_{N0}$ ,  $p_{N1}$ , and so forth, are found.

$$\frac{f(D)}{F(D)} 1 = 1 \left[ \frac{f(0)}{F(0)} - \sum_{\lambda} \frac{f(\lambda)}{\lambda F'(\lambda)} e^{\lambda t} \right]$$

where

$$\frac{f(0)}{F(0)} = p_{N0}$$

and

$$\frac{f(\lambda)}{\lambda F'(\lambda)} = p_{N_1}, p_{N_2}, \text{ and so forth}$$

The expressions  $F(0)$  and  $F'(\lambda)$  are identical for all the equations and vary only with the value of the particular root. As shown in reference 7, they may be found by combining the roots as follows:

$$F(0) = \lambda_1 \lambda_2 \lambda_3 \lambda_4$$

$$F'(\lambda_1) = \lambda_1 (\lambda_1 - \lambda_2) (\lambda_1 - \lambda_3) (\lambda_1 - \lambda_4)$$

$$F'(\lambda_2) = \lambda_2 (\lambda_2 - \lambda_1) (\lambda_2 - \lambda_3) (\lambda_2 - \lambda_4)$$

$$F'(\lambda_3) = \lambda_3 (\lambda_3 - \lambda_1) (\lambda_3 - \lambda_2) (\lambda_3 - \lambda_4)$$

$$F'(\lambda_4) = \lambda_4 (\lambda_4 - \lambda_1) (\lambda_4 - \lambda_2) (\lambda_4 - \lambda_3)$$

The forms of the expressions  $f(0)$  and  $f(\lambda)$  are different for each variable ( $p, r, v$ ) and for each disturbance ( $Y, L, N$ ) and the values of the expressions are different for each root. The nine required equations are listed below. The value of  $f(0)$  may be found by substituting zero for the value of  $\lambda$ , and  $f(\lambda)$  may be found by substituting the appropriate values of  $\lambda$ .

Disturbances	Motion
	p
Y	$(L_r N_\beta - L_\beta N_r) \lambda + L_\beta \lambda^2$
L	$\lambda^3 - \lambda^2 \left( N_r + \frac{Y_\beta}{U_0} \right) + \lambda \left( N_\beta + N_r \frac{Y_\beta}{U_0} \right)$
N	$\left[ -L_\beta - \left( L_r \frac{Y_\beta}{U_0} \right) \right] \lambda + L_r \lambda^2$
	r
	Y
Y	$\lambda^2 N_\beta + \lambda (L_\beta N_p - L_p N_\beta)$
L	$\lambda^2 N_p - \lambda N_p \frac{Y_\beta}{U_0} + N_\beta \frac{g}{U_0}$
N	$\lambda^3 - \lambda^2 \left( \frac{Y_\beta}{U_0} + L_p \right) + \lambda L_p \frac{Y_\beta}{U_0} - L_\beta \frac{g}{U_0}$
	v
	Y
Y	$U_0 \left[ \lambda^3 - \lambda^2 (L_p + N_r) + \lambda (L_p N_r - L_r N_p) \right]$
L	$g(\lambda - N_r) - \lambda N_p$
N	$U_0 \left( \frac{g L_r}{U_0} + \lambda L_p - \lambda^2 \right)$

The equations for  $\phi$  and  $\psi$  may be easily obtained by integrating those for p and r. The  $\phi$  and  $\psi$  thus obtained will be the angles developed in rotation about the X- and Z-axes, respectively. Dividing the equation for v by  $U_0$  gives very closely, the angle of sideslip  $\beta$ .

When the stability equation yields a pair of conjugate complex roots, the necessary computations are more easily made if the expressions involving these roots are combined. The example previously cited then changes to this form:

$$p_N(t) = p_{N_0} + p_{N_1} e^{\lambda_1 t} + p_{N_2} e^{\lambda_2 t} + 2\sqrt{I^2 + J^2} e^{at} \cos b(t + t_{p_N})$$

where

$$I + iJ = p_{N_3}$$

a real part of the complex root

b imaginary part of the complex root

$t_{p_N}$  phase shift for the damped cosine curve and equal to

$$(1/b) \tan^{-1} \left( \frac{J}{I} \right)$$

To convert the motion of the airplane relative to the air to motion relative to the ground, the following equations were used:

$$\text{Lateral distance } X = \int [U_0 \sin \psi + v_0 + v \cos \psi] dt$$

$$\text{Longitudinal distance } Y = \int [U_0 \cos \psi - v \sin \psi] dt$$

where  $v_0$  is the velocity of the airplane as carried with the gust.

## REFERENCES

1. Pearson, Henry A., and Jones, Robert T.: Theoretical Stability and Control Characteristics of Wings with Various Amounts of Taper and Twist. NACA Rep. No. 635, 1938.
2. Jones, Robert T.: The Influence of Lateral Stability on Disturbed Motions of an Airplane with Special Reference to the Motions Produced by Gusts. NACA Rep. No. 638, 1938.
3. Jones, Robert T.: Calculation of the Motion of an Airplane under the Influence of Irregular Disturbances. Jour. Aero. Sci., vol. 3, no. 12, Oct. 1936, pp. 419-425.
4. Jones, Robert T.: A Simplified Application of the Method of Operators to the Calculation of Disturbed Motions of an Airplane. NACA Rep. No. 560, 1936.
5. Jones, Robert T.: A Study of the Two-Control Operation of an Airplane. NACA Rep. No. 579, 1936.
6. Gilruth, R. R.: Requirements for Satisfactory Flying Qualities of Airplanes. NACA ACR, April 1941. (Classification changed to "Restricted," Oct. 1943.)
7. Donlan, C. J., and Recant, I. G.: Methods of Analyzing Wind-Tunnel Data for Dynamic Flight Conditions. NACA TN No. 828, 1941.

TABLE I.- THE PHYSICAL PROPERTIES OF  
THE AIRPLANES INVESTIGATED

Properties	Airplane 1	Airplane 2	Airplane 3	Airplane 4
S	2510	1800.0	275	296
b	173.25	134.0	40	39
m	3880	2,795	261	217.5
l	59.22	-----	19.08	-----
$I_{xx}$	1,959,627	666,121	4,584	9,800
$I_{zz}$	3,152,174	869,060.9	14,597	10,900
$\rho$	.002378	.002378	.001189	.001189
$U_0$	264.5	264.5	626	626
$C_L$	.600	.601	.132	.102
$\alpha$	.1054	.1054	.0232	.0179
$C_{Y\beta}$	-.013	-.005	-.0105	-.0055
$C_{l\beta}$	-.0016	-.0015	-.0012	-.0009
$C_{n\beta}$	.0022	.0004	.0016	.0007
$C_{lr}$	.1202	.11333	.0244	.01718
$C_{nrwing}$	-.0051	-.00357	-.0100	-.0002
$C_{nr tail}$	-.1175	-.0050 -.0500 -.0150	-.1257	-.0050 -.0500 -.0150
$C_{lp}$	-.542	-.515	-.452	-.442
$C_{np}$	-.0406	-.0382	-.0058	-.0045

TABLE III.- THE STABILITY QUARTIC ROOTS, RATES OF DAMPING AND  
PERIODS FOR THE AIRPLANES CONSIDERED

Item	Airplane 1	Airplane 2			Airplane 3	Airplane 4		
		Case a	Case b	Case c		Case a	Case b	Case c
$\lambda_1$	0.0060	0.0120	0.0053	-0.0181	-0.0088	0.0012	-0.0003	-0.0054
$^a t_{1/10}$	15.74	7.93	18.04	5.8	11.95	80.4	382.5	19.6
$\lambda_2$	-3.2888	-3.9391	-3.9389	-3.9383	-8.0737	-3.8087	-3.8096	-3.8096
$^b t_{1/2}$	.21	.18	.18	.18	.09	.18	.18	.18
$\alpha$	-.3030	-.0552	-.0811	-.1721	-.4710	-.0739	-.1121	-.2434
$^b t_{1/2}$	2.3	12.56	8.54	4.03	1.47	9.37	6.18	2.85
$b$	1.2537	.8334	.8324	.8189	.40333	3.1222	3.1230	3.1197
$^c \frac{2\pi}{b}$	5.02	7.54	7.55	7.67	1.56	2.01	2.01	2.01

<sup>a</sup>Time for amplitude to increase or diminish by one-tenth.<sup>b</sup>Time for amplitude to diminish by one-half.<sup>c</sup>Period of oscillation.

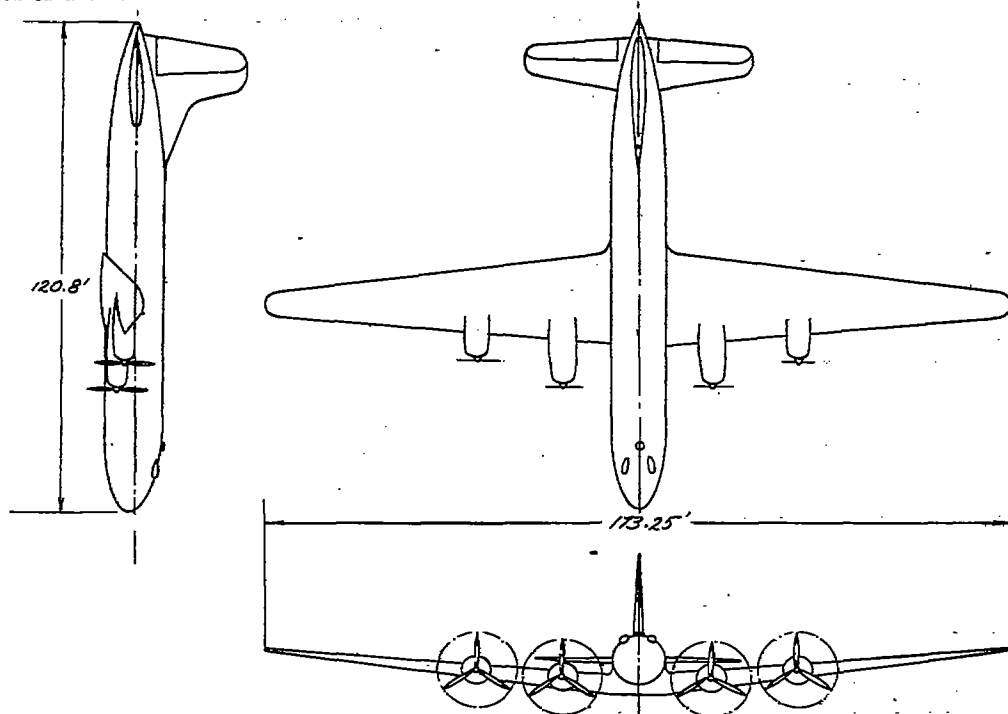
TABLE II  
CONSTANTS FOR EQUATIONS OF MOTION

BANK

YAW

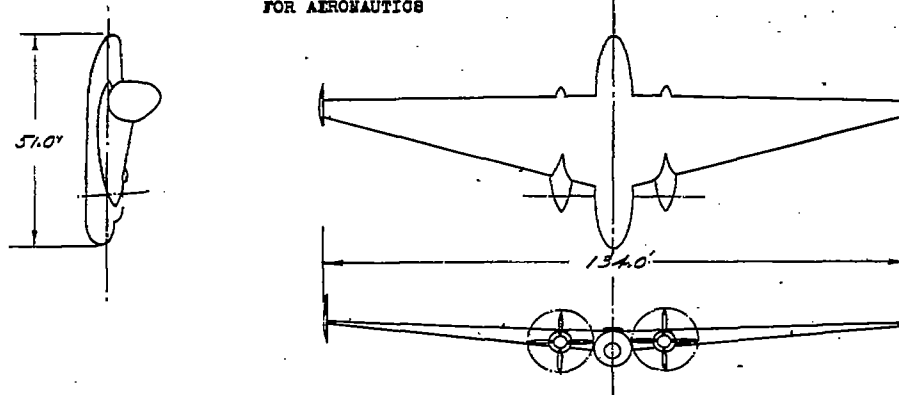
SIDESLIP

	BANK					YAW					SIDESLIP				
	$\phi_{10}$	$\phi_{11}$	$\phi_{12}$	$(2I^2+J^2)_{\phi_1}$	$t_{\phi_1}$	$\psi_{10}$	$\psi_{11}$	$\psi_{12}$	$(2I^2+J^2)_{\psi_1}$	$t_{\psi_1}$	$\beta_{10}$	$\beta_{11}$	$\beta_{12}$	$(2I^2+J^2)_{\beta_1}$	$t_{\beta_1}$
AIRPLANE 1	-15.91786	15.84442	0.00081	0.02209	5.08009	-5.35252	879.2227	0.00874	0.04793	-0.72112	-1.63653	174159	-0.00744	0.05101	1.04939
AIRPLANE 2 CASE "a"	-14.14378	14.11473	.06334	.10020	5.23419	-1.71110	14.55980	.00349	.10953	-1.45352	-1.0523	.80914	-.00351	.12446	3.35993
CASE "b"	-32.34290	32.32124	.06335	.10028	5.14616	-3.83453	7347054	.00352	.10997	-5.0223	-.81119	1.03369	-.00352	.12604	3.33969
CASE "c"	9.62411	-9.62117	.06335	.10215	4.88245	1.12737	6317843	.00371	.11814	-.68437	.76286	-.64618	-.00376	.11872	3.15630
AIRPLANE 3	15.85114	-15.84571	.01521	.00046	.85733	.70368	30.15409	.00003	.00053	-.18990	.03323	-.03290	-.00013	.00059	.60125
AIRPLANE 4 CASE "a"	-217.86044	217.79424	.06723	.00165	1.04632	-11.18399	3466.5889	-.00001	.00173	1.25255	-.04625	.04354	-.00094	.00180	.77306
CASE "b"	387.61839	-387.65489	.06780	.00166	1.03901	43.13624	17532.0657	-.00001	.00179	-2.5631	.58367	-.53141	-.00094	.00181	.76273
CASE "c"	40.18695	-40.25306	.06734	.00173	.98501	2.46294	46.74485	-.00001	.00182	-.23110	.03973	-.03665	-.00094	.00185	.72860
	DUE TO ROLLING DISTURBANCE					DUE TO YAWING DISTURBANCE									
	$\phi_{10}$	$\phi_{11}$	$\phi_{12}$	$(2I^2+J^2)_{\phi_1}$	$t_{\phi_1}$	$\psi_{10}$	$\psi_{11}$	$\psi_{12}$	$(2I^2+J^2)_{\psi_1}$	$t_{\psi_1}$	$\beta_{10}$	$\beta_{11}$	$\beta_{12}$	$(2I^2+J^2)_{\beta_1}$	$t_{\beta_1}$
AIRPLANE 1	54.57772	54.48925	-0.00517	0.27000	0.97056	-6.23838	1446.00440	-0.00221	0.58502	2.24300	-2.67839	2.07000	0.00042	0.62345	-0.17851
AIRPLANE 2 CASE "a"	-80.11294	80.01542	-.00312	1.16315	1.73078	-3.64746	302.4962	-.00017	1.27210	3.61819	-3.18547	1.73851	.00027	1.4446	-.03918
CASE "b"	-184.87149	184.71280	-.00317	1.16023	1.71905	-21.77934	412.53309	-.00015	1.27305	3.61854	-7.25477	5.21181	.00028	1.44630	-.03745
CASE "c"	62.78230	-53.16063	-.00334	1.17653	1.51530	6.30357	34903507	-.00020	1.30305	3.61160	2.10543	-3.67637	-.00030	1.43266	-.18436
AIRPLANE 3	33.57310	-33.62313	.00742	.06339	2.2243	1.92246	134.64764	.00002	.05933	.73970	.09302	-.07842	-.00006	.06084	-.02920
AIRPLANE 4 CASE "a"	131.90630	-131.80663	.03849	.00300	.27541	-16.11943	1163035239	-.00013	.00660	.38745	-17.113	.06903	-.00062	.10180	-.00543
CASE "b"	1349.26231	-1349.36492	.03863	.00337	.26605	69.23674	85862.0623	-.00001	.10045	.90353	.73857	-.83663	-.00064	.10169	-.00310
CASE "c"	62.03635	-62.14700	.04067	.09508	.23437	3.64509	66.125357	-.00001	.10031	.27227	.03764	-.13830	-.00062	.10167	-.02204
	DUE TO ROLLING DISTURBANCE					DUE TO YAWING DISTURBANCE									
	$\phi_{10}$	$\phi_{11}$	$\phi_{12}$	$(2I^2+J^2)_{\phi_1}$	$t_{\phi_1}$	$\psi_{10}$	$\psi_{11}$	$\psi_{12}$	$(2I^2+J^2)_{\psi_1}$	$t_{\psi_1}$	$\beta_{10}$	$\beta_{11}$	$\beta_{12}$	$(2I^2+J^2)_{\beta_1}$	$t_{\beta_1}$
AIRPLANE 1	-0.03186	0.02977	0.00019	0.00126	-0.41663	-0.57217	0.57085	0.00001	0.00274	0.85586	0	0.00113	0.00002	0.00232	3.44636
AIRPLANE 2 CASE "a"	-.03106	.02735	.00016	.00339	-.30226	-.23511	.23414	.00001	.00393	1.64935	0	.00060	-.00001	.00446	5.19630
CASE "b"	-.03106	.02743	.00016	.00358	-.31414	-.63456	.63357	.00001	.00393	1.58636	0	.00083	-.00001	.00447	5.12787
CASE "c"	-.03106	.02787	.00016	.00364	-.73123	.18126	-.18300	.00001	.00403	1.37503	0	.00137	-.00001	.00459	5.24620
AIRPLANE 3	-.03106	.03063	.00010	.00044	-.18430	.17725	-.17934	.00000	.00339	.33330	0	.00007	.00000	.00039	1.12301
AIRPLANE 4 CASE "a"	-.03106	.03045	.00025	.00046	-.22432	-1.30461	1.32459	.00000	.00050	.48732	0	.00001	.00000	.00160	1.50916
CASE "b"	-.03106	.03047	.00025	.00047	-.23732	5.69303	-5.69362	.00000	.00050	.48007	0	.00002	.00000	.00051	1.49911
CASE "c"	-.03106	.03042	.00024	.00047	-.23372	.29432	-.29439	.00000	.00050	.16414	0	.00006	.00000	.00051	1.47387



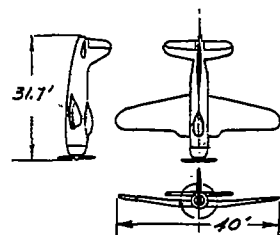
AIRPLANE 1

NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

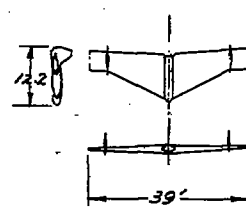


AIRPLANE 2

CASE 'a',  $C_{L_0} = -.005$   
CASE 'b',  $C_{L_0} = -.015$   
CASE 'c',  $C_{L_0} = -.050$



AIRPLANE 3



AIRPLANE 4

CASE 'a',  $C_{L_0} = -.005$   
CASE 'b',  $C_{L_0} = -.015$   
CASE 'c',  $C_{L_0} = -.050$

FIGURE 1.- THE RELATIVE SIZES AND FORMS OF THE  
FOUR AIRPLANES INVESTIGATED

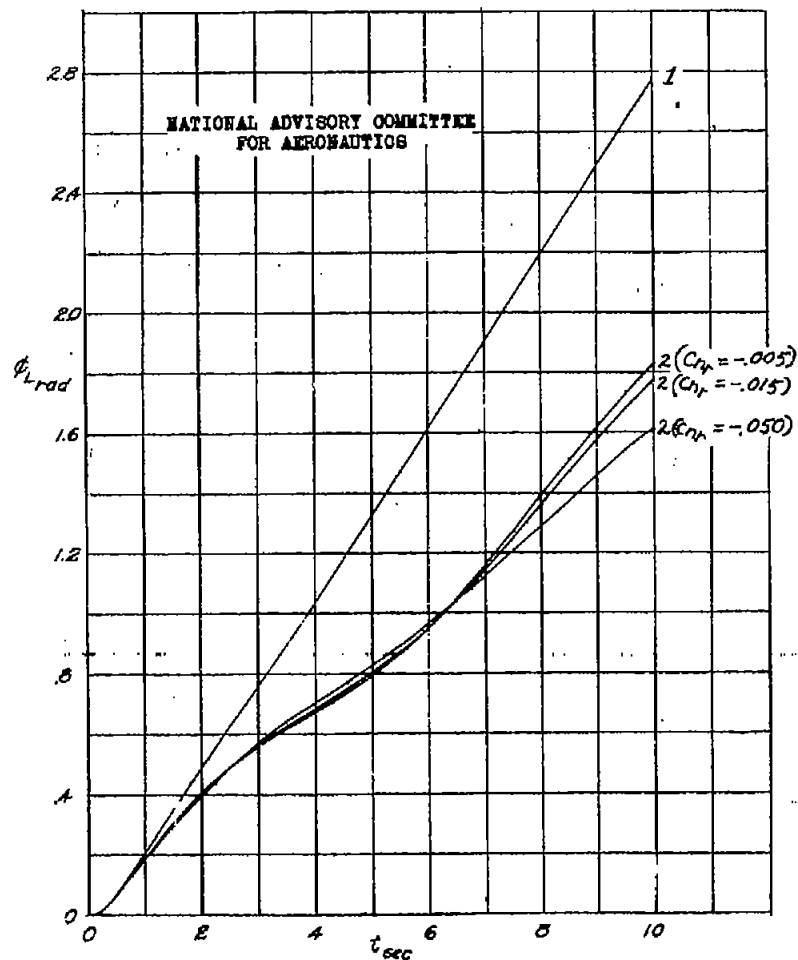


FIGURE 2.- THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE APPLICATION OF A UNIT ROLLING ACCELERATION TO AIRPLANES 1 AND 2.

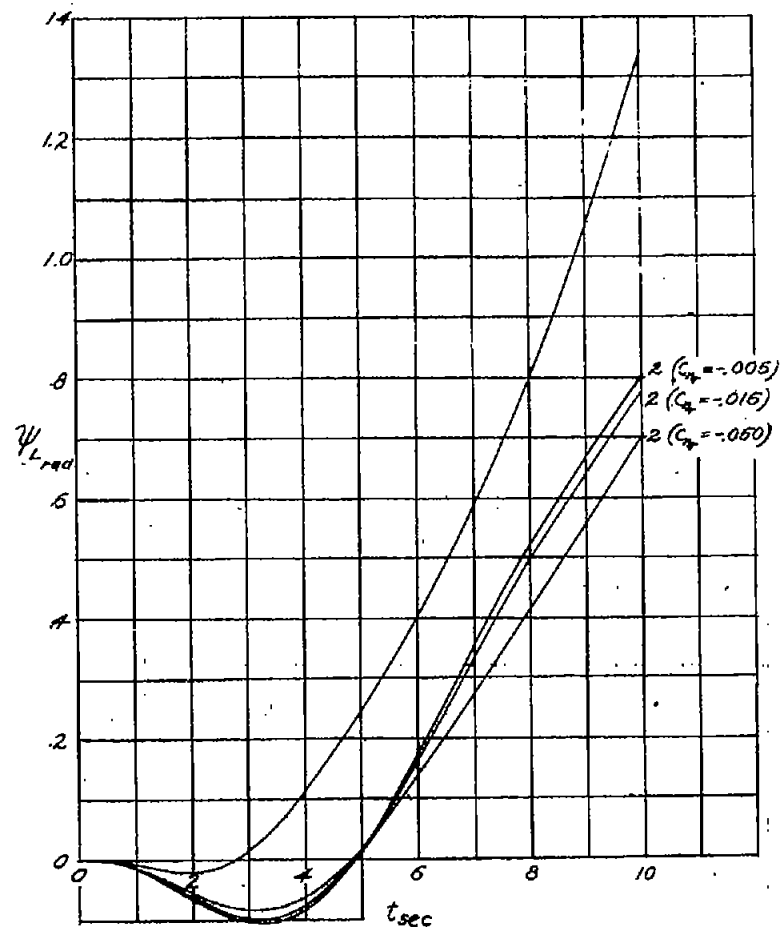


FIGURE 3.- THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE APPLICATION OF A UNIT ROLLING ACCELERATION TO AIRPLANES 1 AND 2.

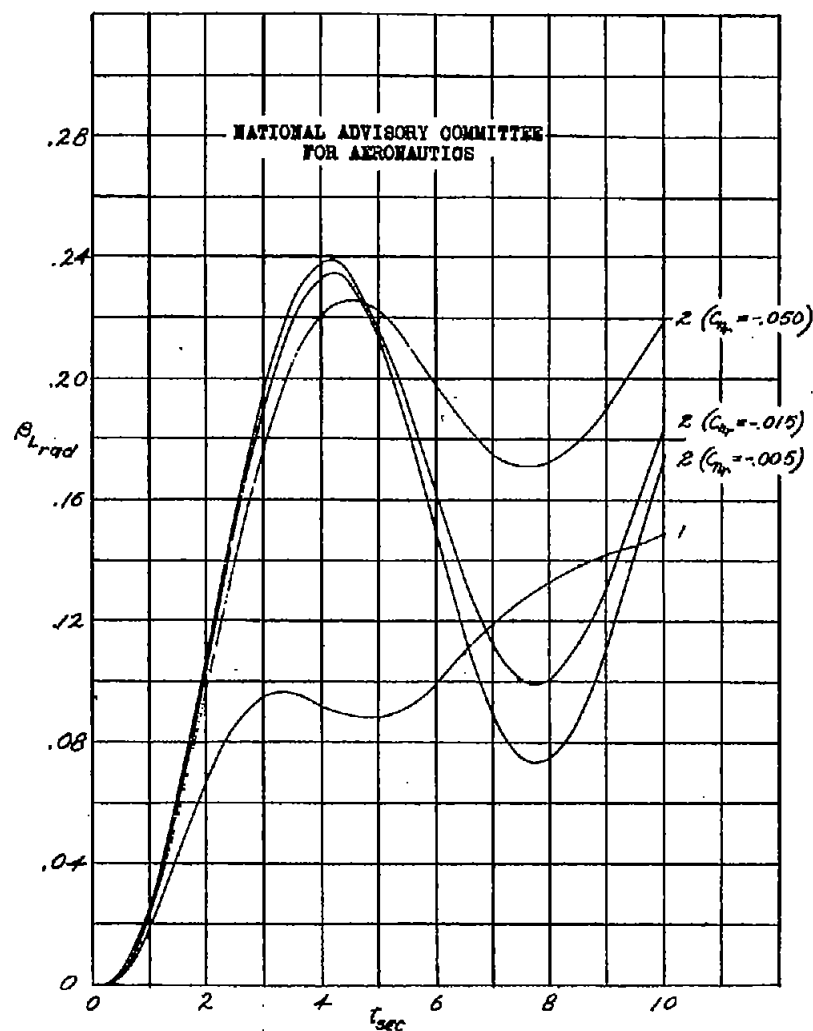


FIGURE 4.- THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE APPLICATION OF A UNIT ROLLING ACCELERATION TO AIRPLANES 1 AND 2.

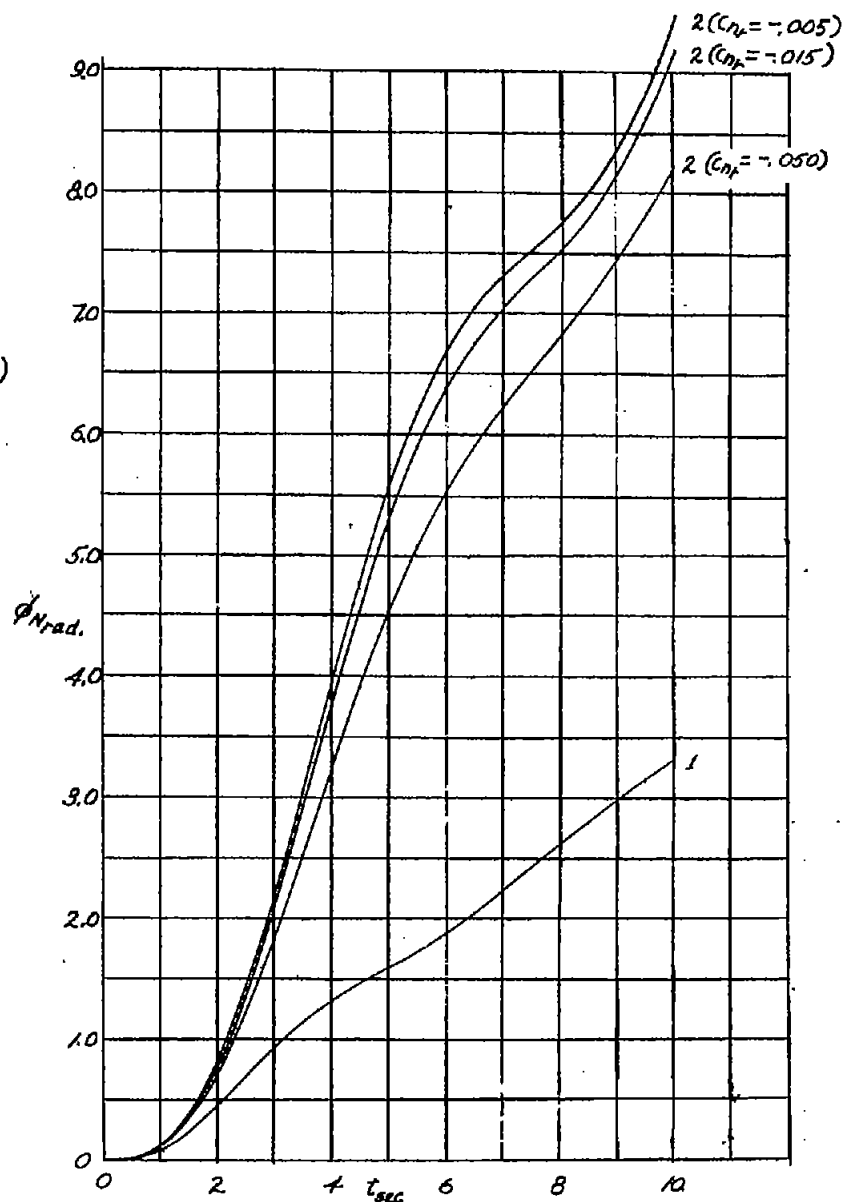


FIGURE 5.- THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE APPLICATION OF A UNIT YAWING ACCELERATION TO AIRPLANES 1 AND 2.

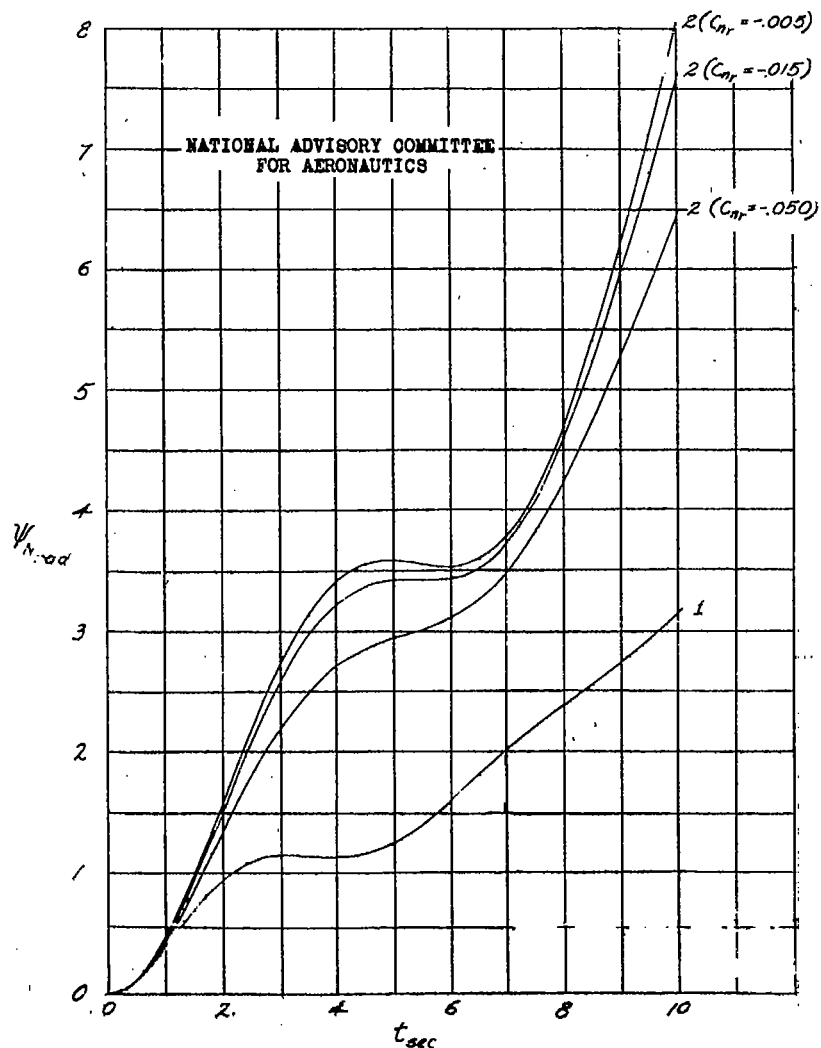


FIGURE 6.- THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE APPLICATION OF A UNIT YAWING ACCELERATION TO AIRPLANES 1 AND 2.

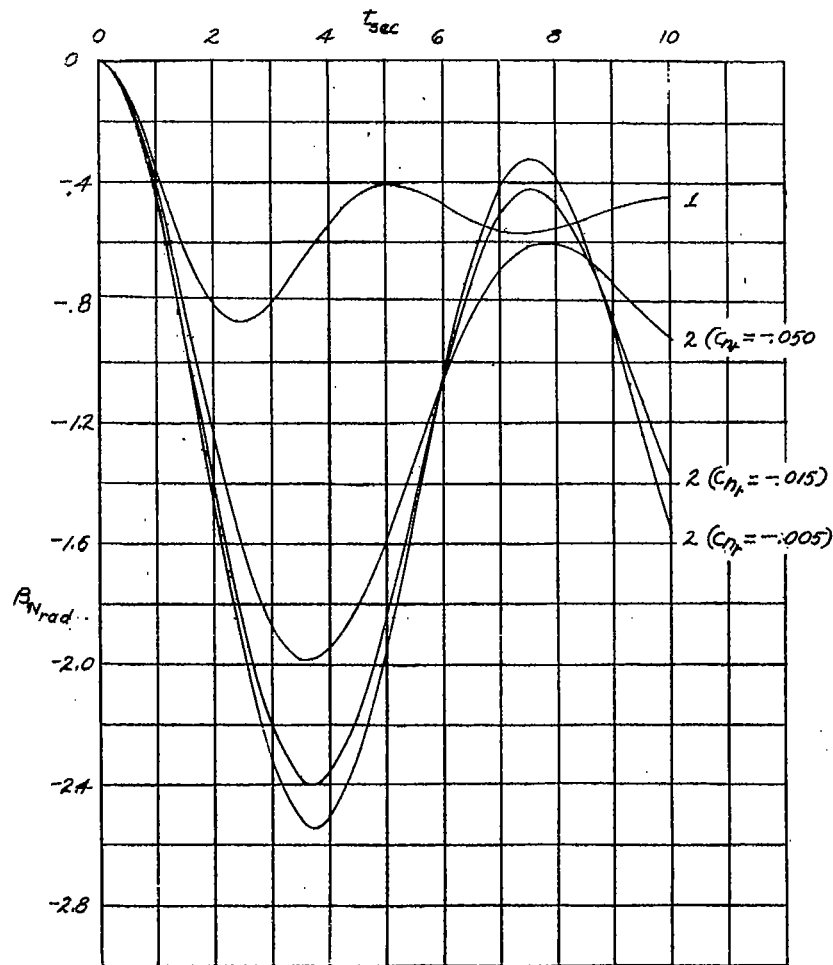


FIGURE 7.- THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE APPLICATION OF A UNIT YAWING ACCELERATION TO AIRPLANES 1 AND 2.

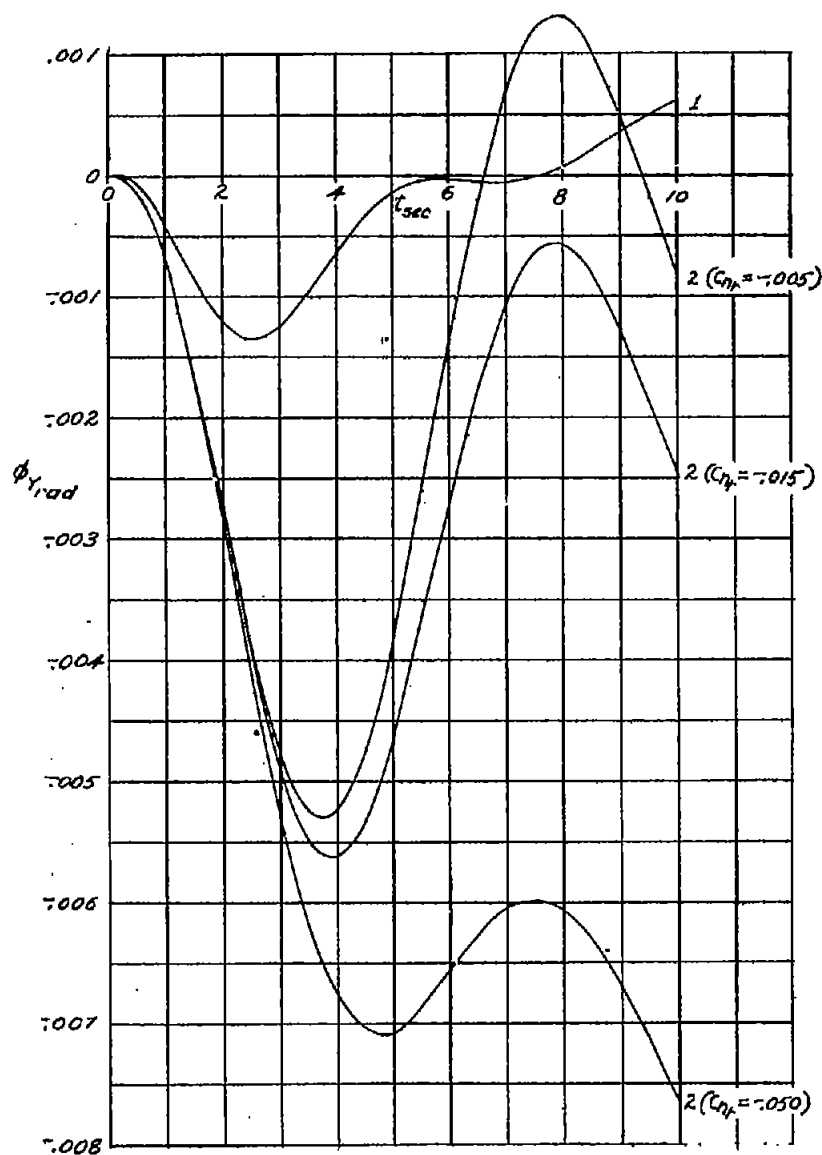


FIGURE 8.- THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE APPLICATION OF A UNIT SIDE ACCELERATION TO AIRPLANES 1 AND 2.

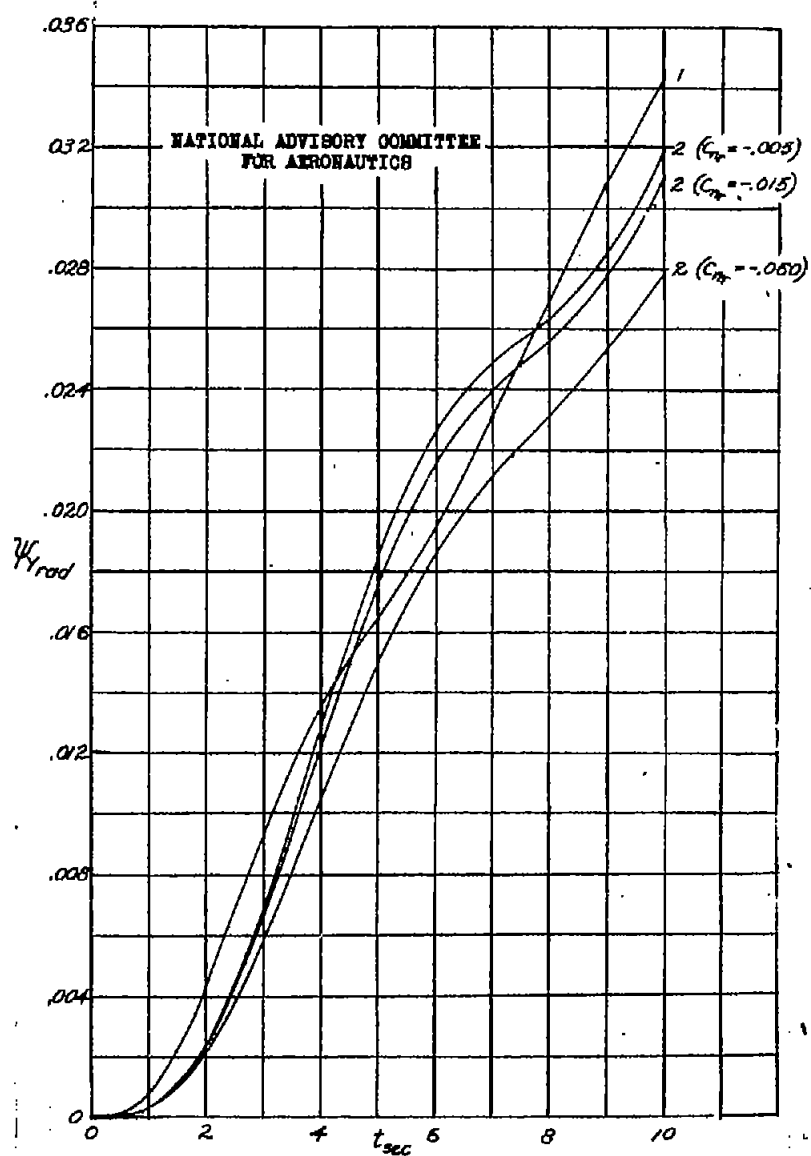


FIGURE 9.- THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE APPLICATION OF A UNIT SIDE ACCELERATION TO AIRPLANES 1 AND 2.

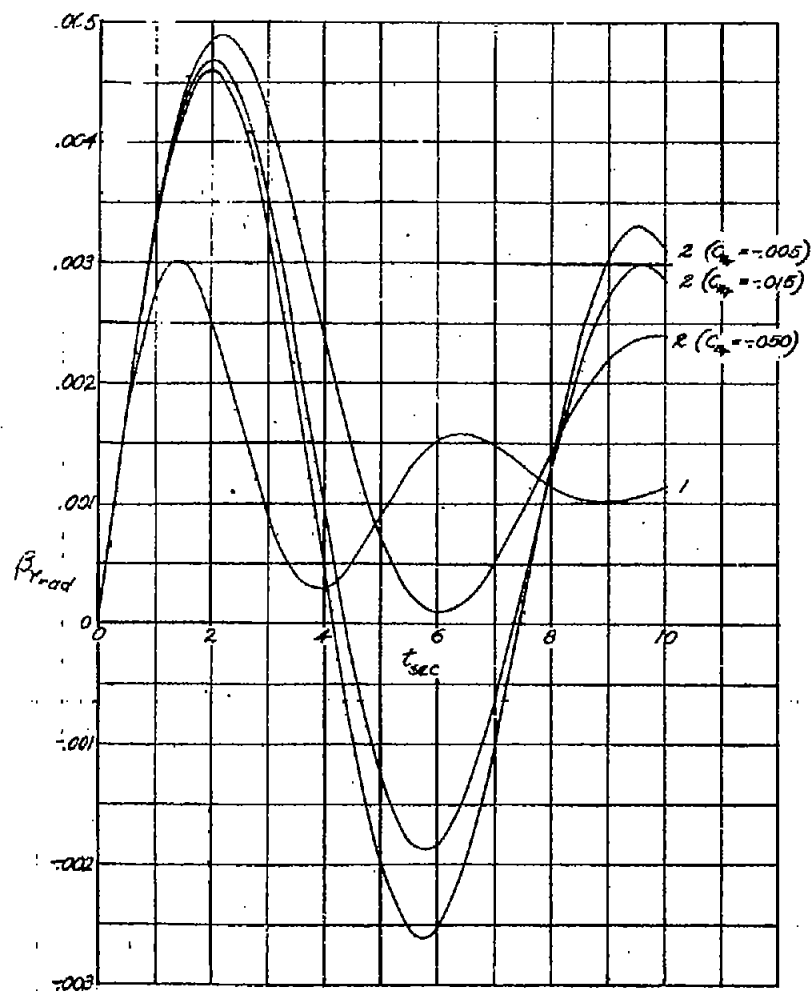


FIGURE 10. - THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE APPLICATION OF A UNIT SIDE ACCELERATION TO AIRPLANES 1 AND 2.

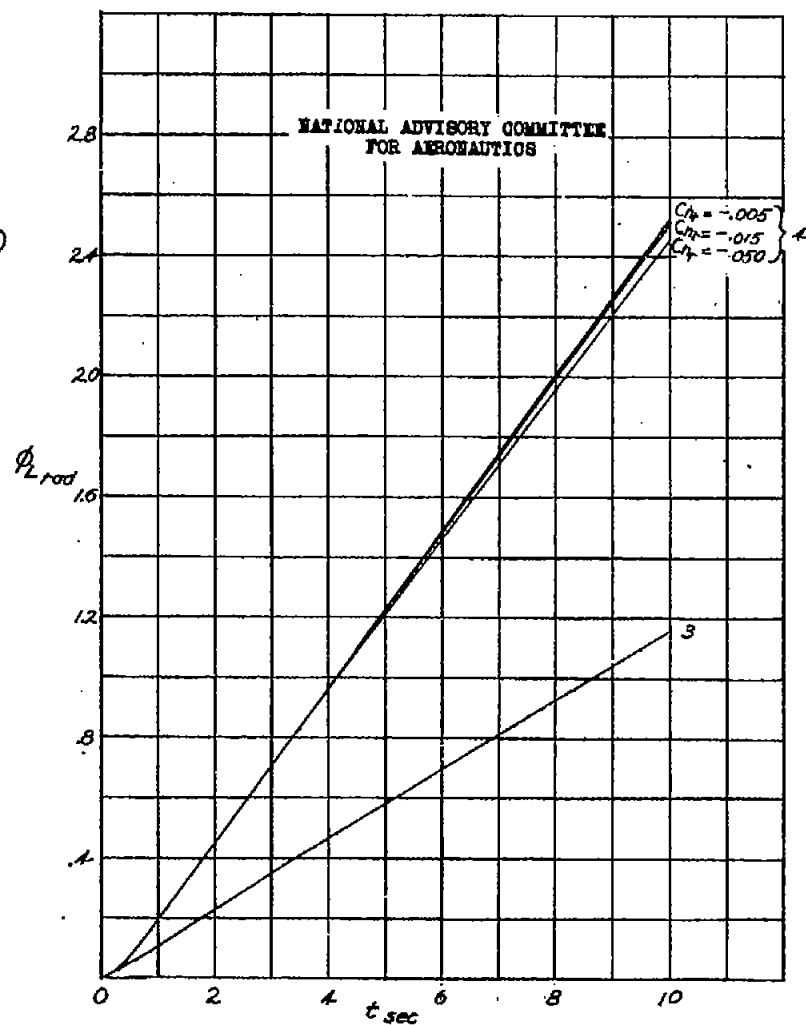


FIGURE 11. - THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE APPLICATION OF A UNIT ROLLING ACCELERATION TO AIRPLANES 3 AND 4.

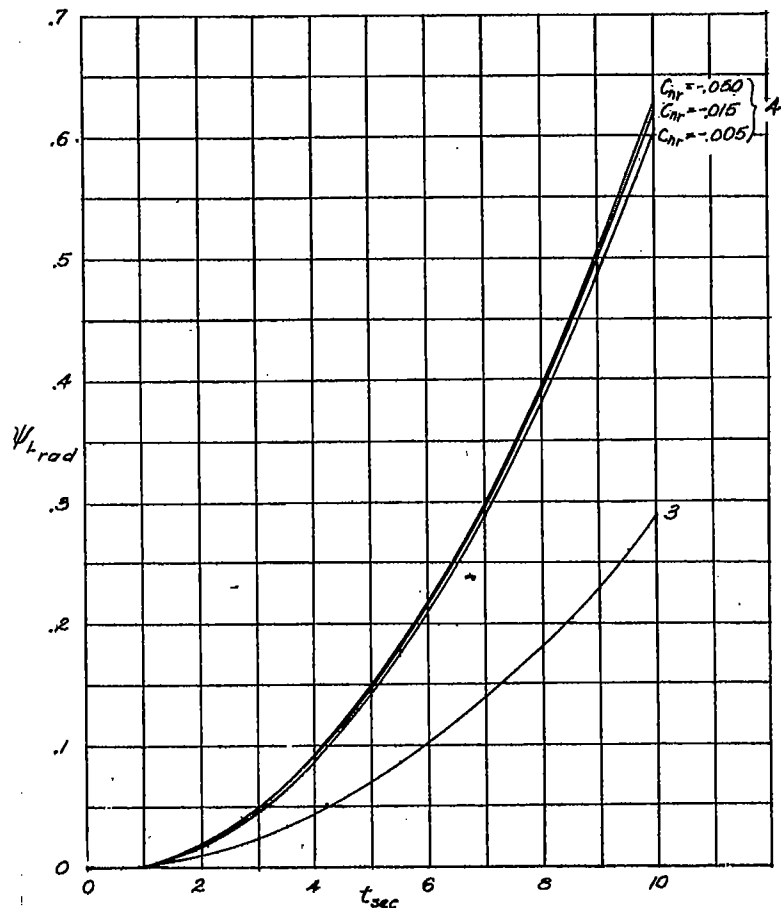


FIGURE 12.- THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE APPLICATION OF A UNIT ROLLING ACCELERATION TO AIRPLANES 3 AND 4..

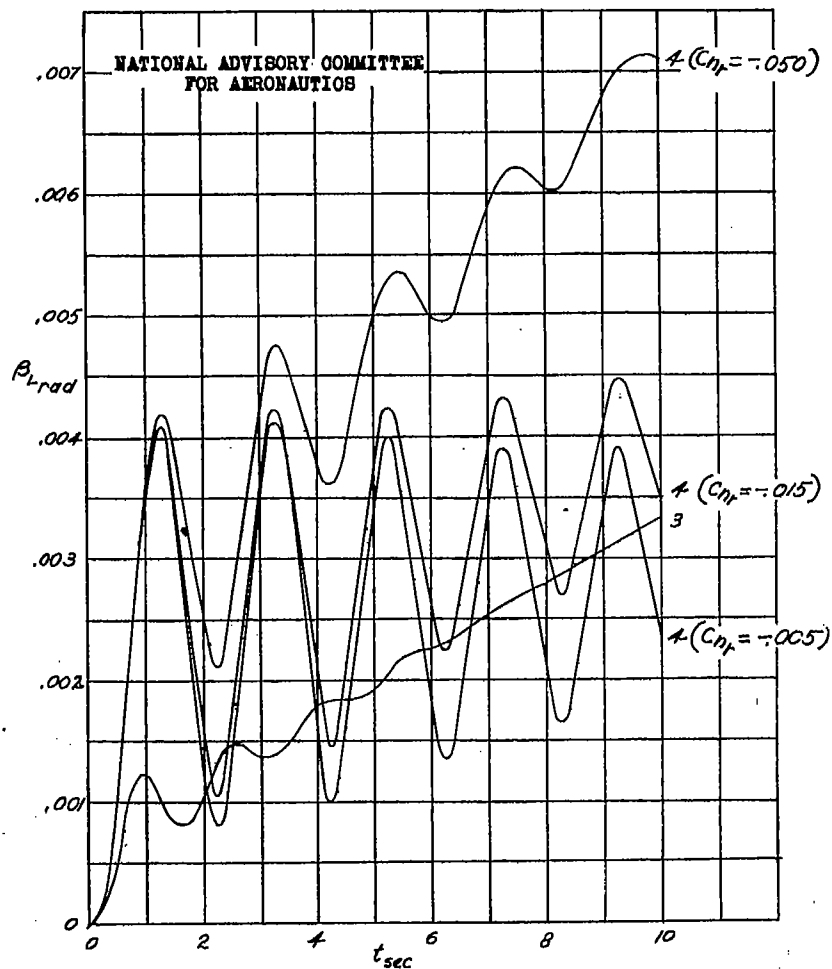


FIGURE 13.- THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE APPLICATION OF A UNIT ROLLING ACCELERATION TO AIRPLANES 3 AND 4.

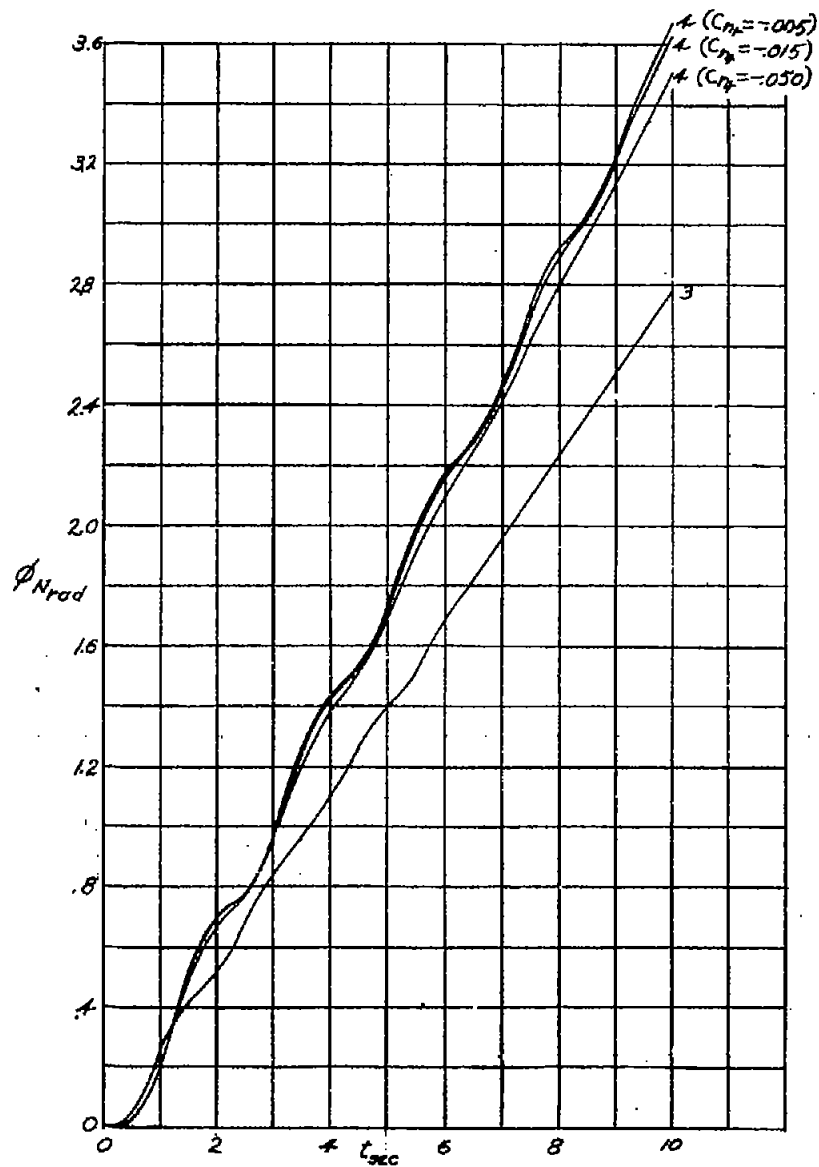


FIGURE 14.—THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE APPLICATION OF A UNIT YAWING ACCELERATION TO AIRPLANES 3 AND 4.

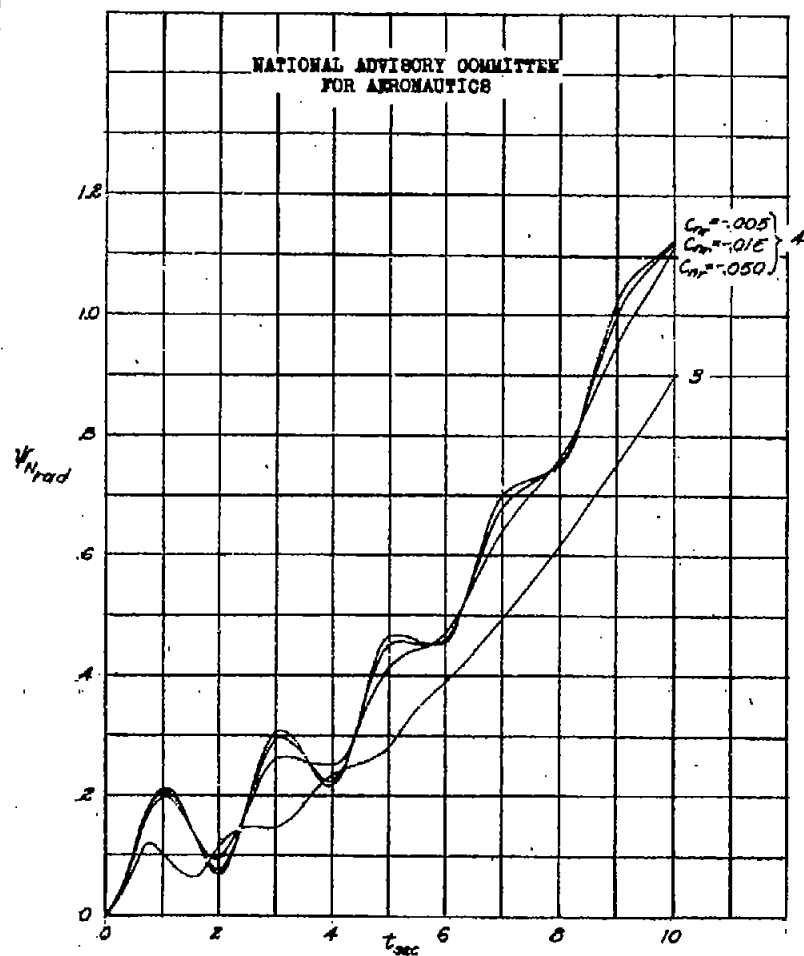


FIGURE 15.—THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE APPLICATION OF A UNIT YAWING ACCELERATION TO AIRPLANES 3 AND 4.

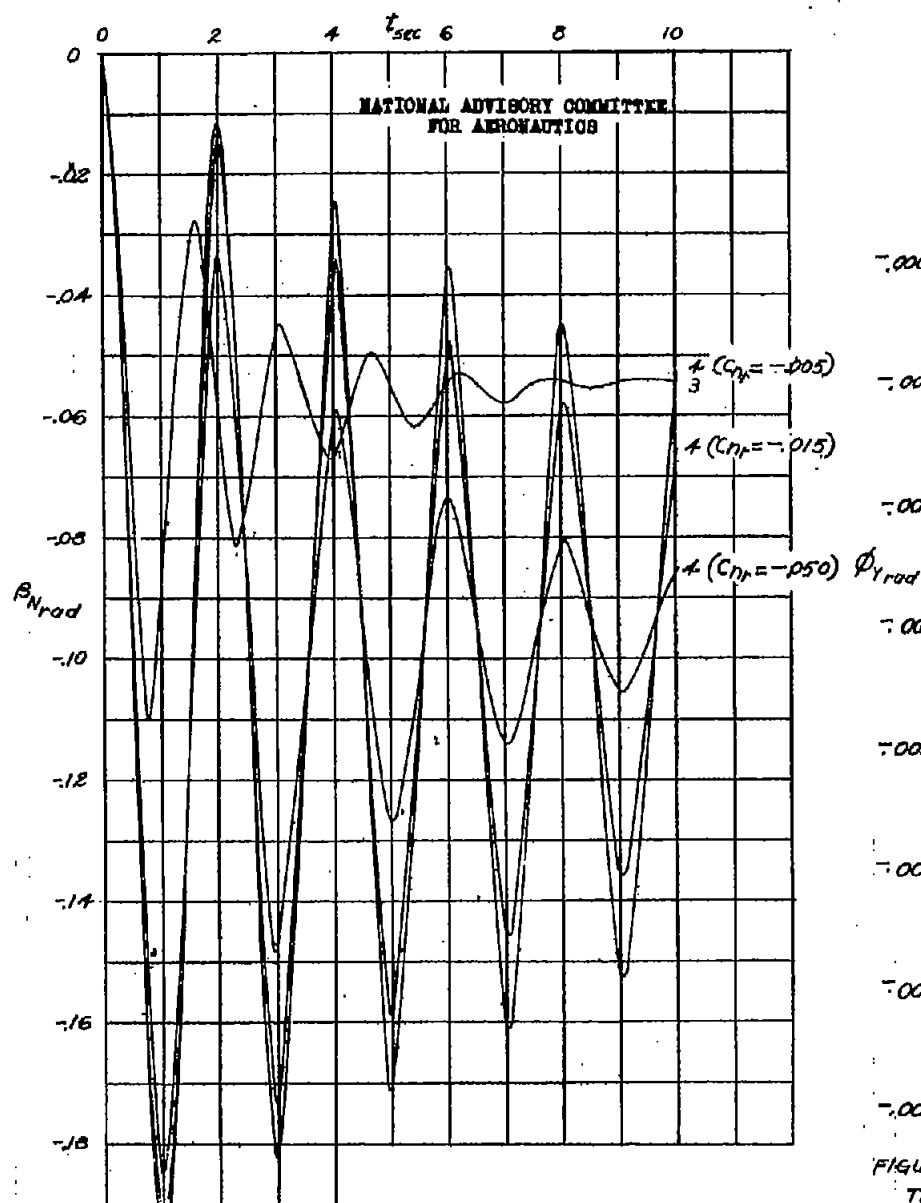


FIGURE 16.- THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE APPLICATION OF A UNIT YAWING ACCELERATION TO AIRPLANES 3 AND 4

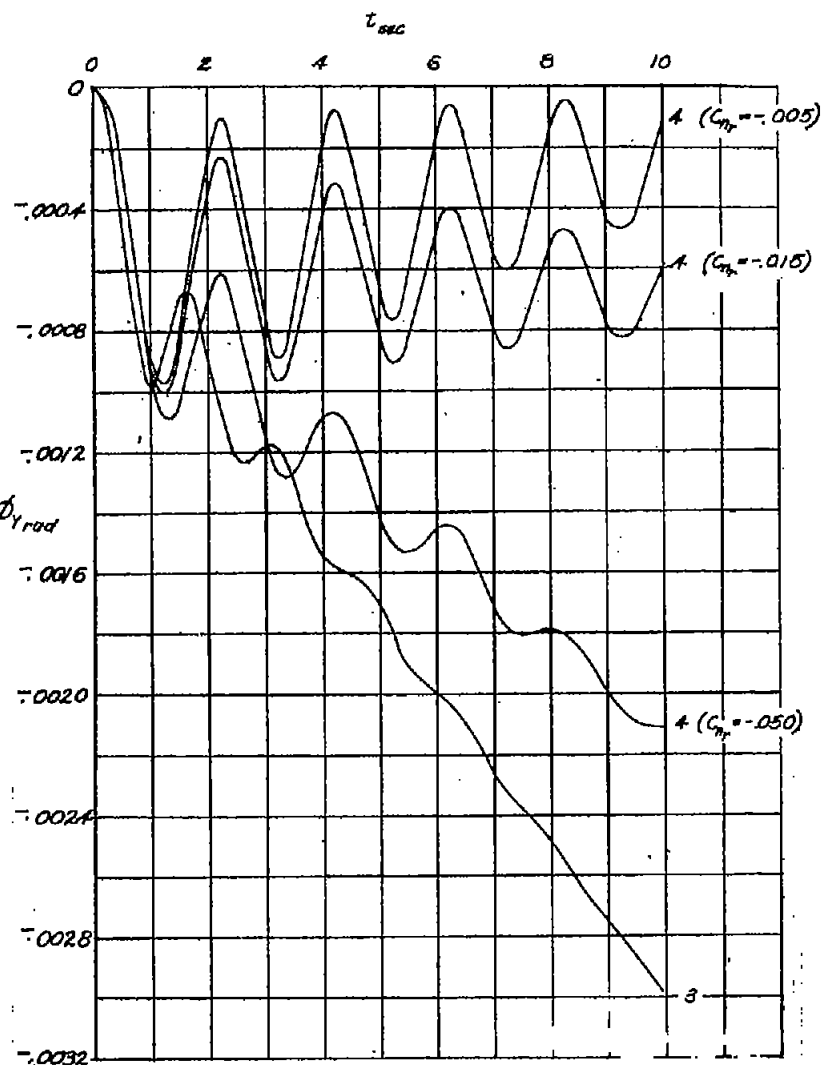


FIGURE 17.- THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE APPLICATION OF A UNIT SIDE ACCELERATION TO AIRPLANES 3 AND 4

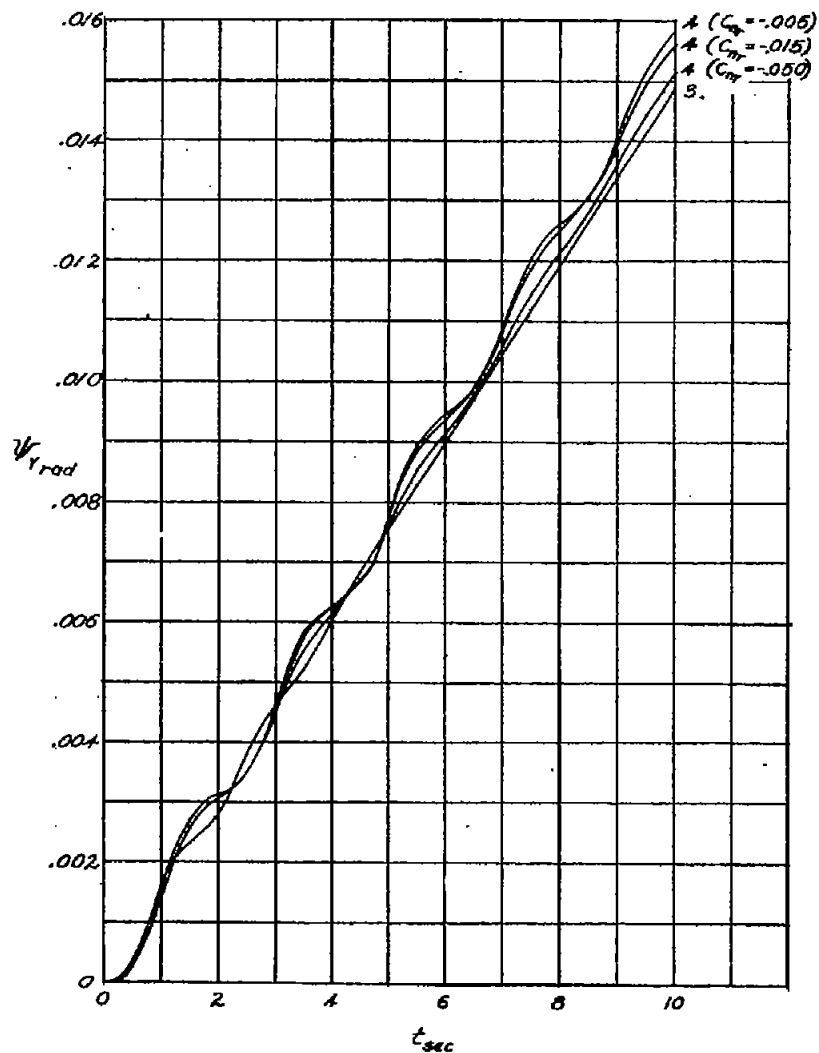


FIGURE 18.- THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE APPLICATION OF A UNIT SIDE ACCELERATION TO AIRPLANES 3 AND 4.

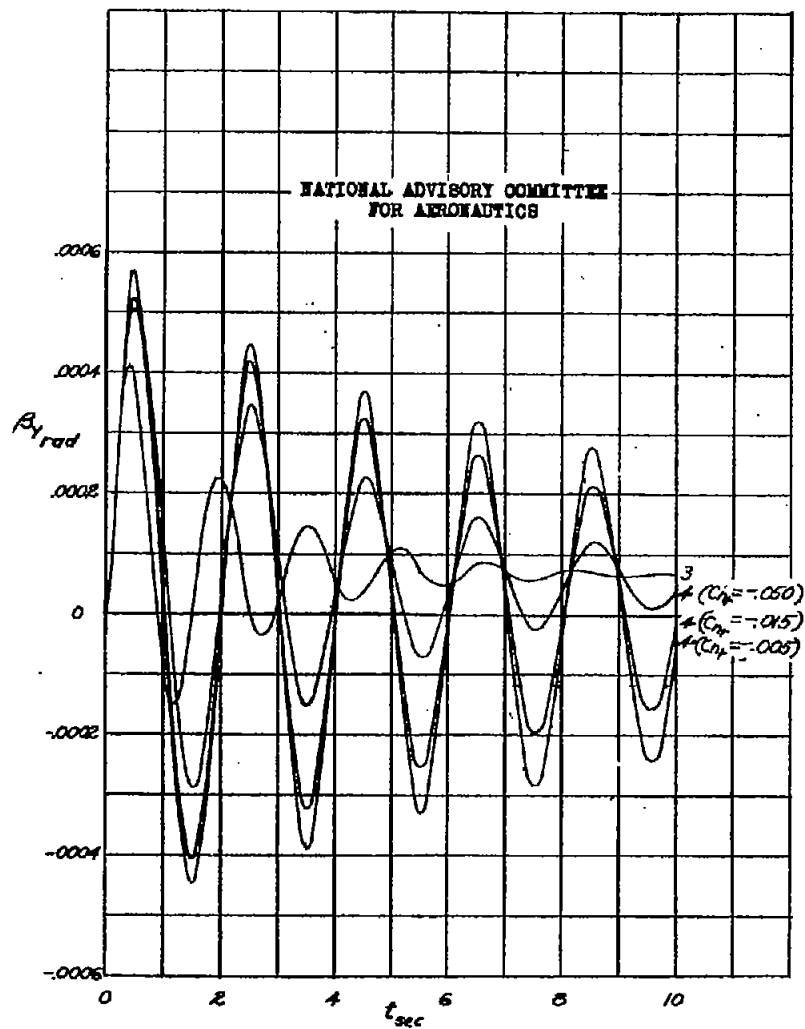


FIGURE 19.- THE VARIATION OF THE ANGLE SIDESLIP WITH TIME DUE TO THE APPLICATION OF A UNIT SIDE ACCELERATION TO AIRPLANES 3 AND 4.

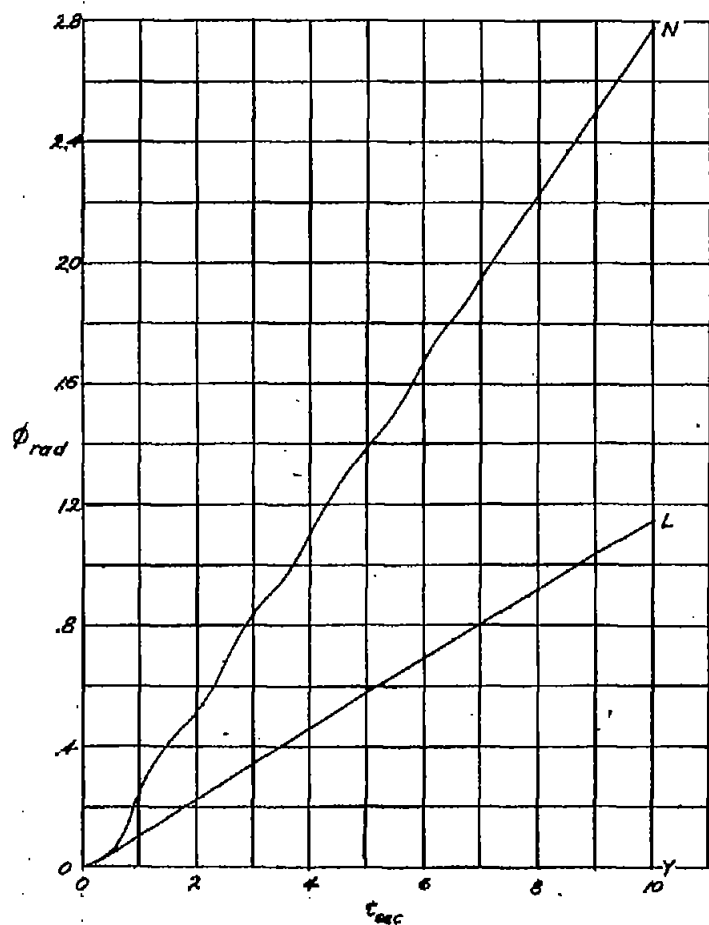


FIGURE 20.-THE VARIATION OF ANGLE OF BANK WITH TIME FOR AIRPLANE 3 SHOWING THE RELATIVE EFFECTS OF A UNIT YAWING ACCELERATION, ROLLING ACCELERATION, AND SIDE ACCELERATION.

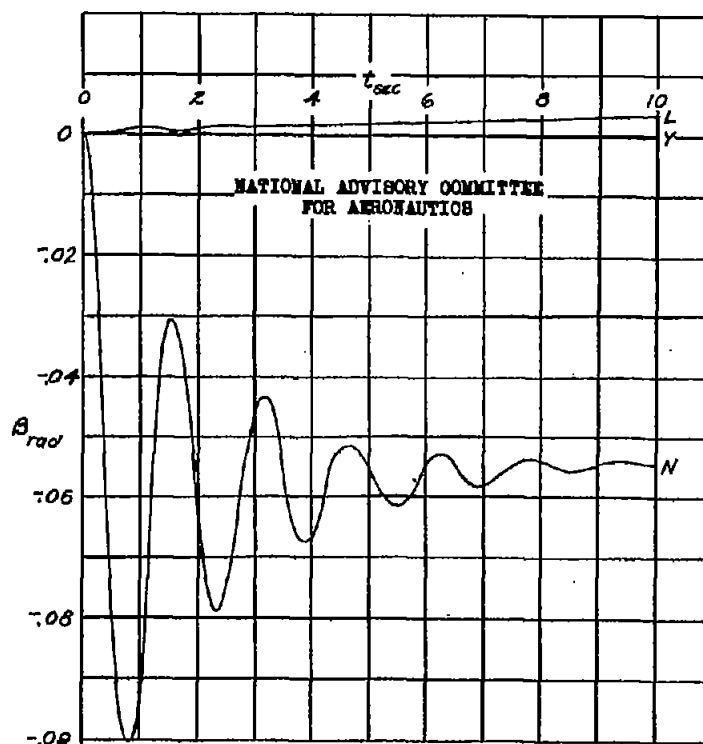


FIGURE 21.-THE VARIATION OF ANGLE OF SIDESLIP WITH TIME FOR AIRPLANE 3 SHOWING THE RELATIVE EFFECTS OF A UNIT YAWING ACCELERATION, ROLLING ACCELERATION, AND SIDE ACCELERATION.

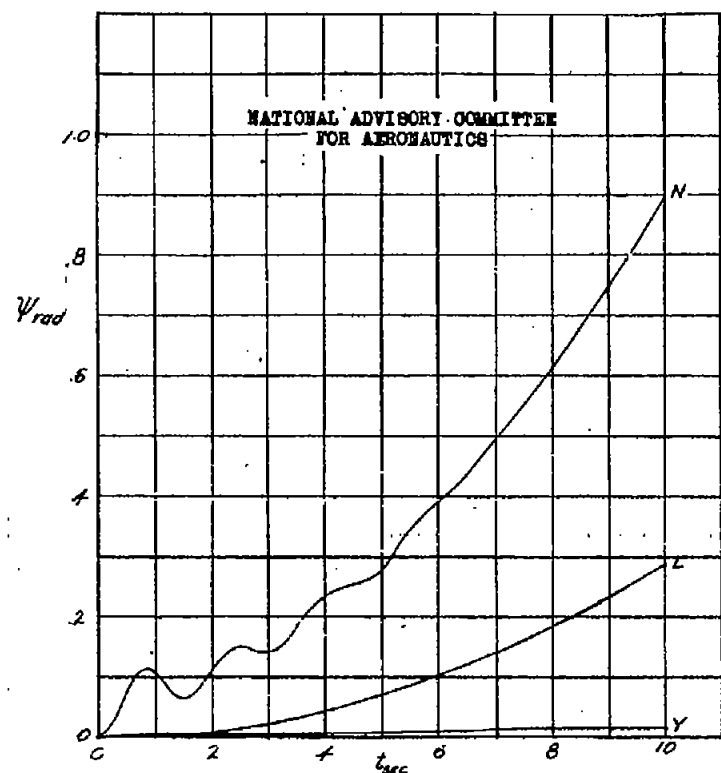


FIGURE 22. - THE VARIATION OF ANGLE OF YAW WITH TIME FOR AIRPLANE 3 SHOWING THE RELATIVE EFFECTS OF A UNIT YAWING ACCELERATION, ROLLING ACCELERATION, AND SIDE ACCELERATION.

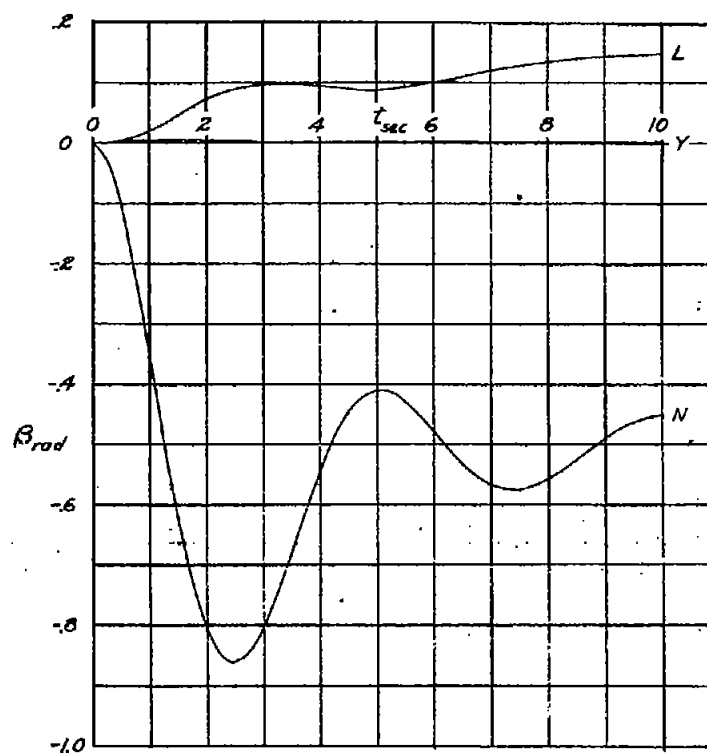


FIGURE 23. - THE VARIATION OF ANGLE OF SIDESLIP WITH TIME FOR AIRPLANE 1 SHOWING THE RELATIVE EFFECTS OF A UNIT YAWING ACCELERATION, ROLLING ACCELERATION, AND SIDE ACCELERATION.

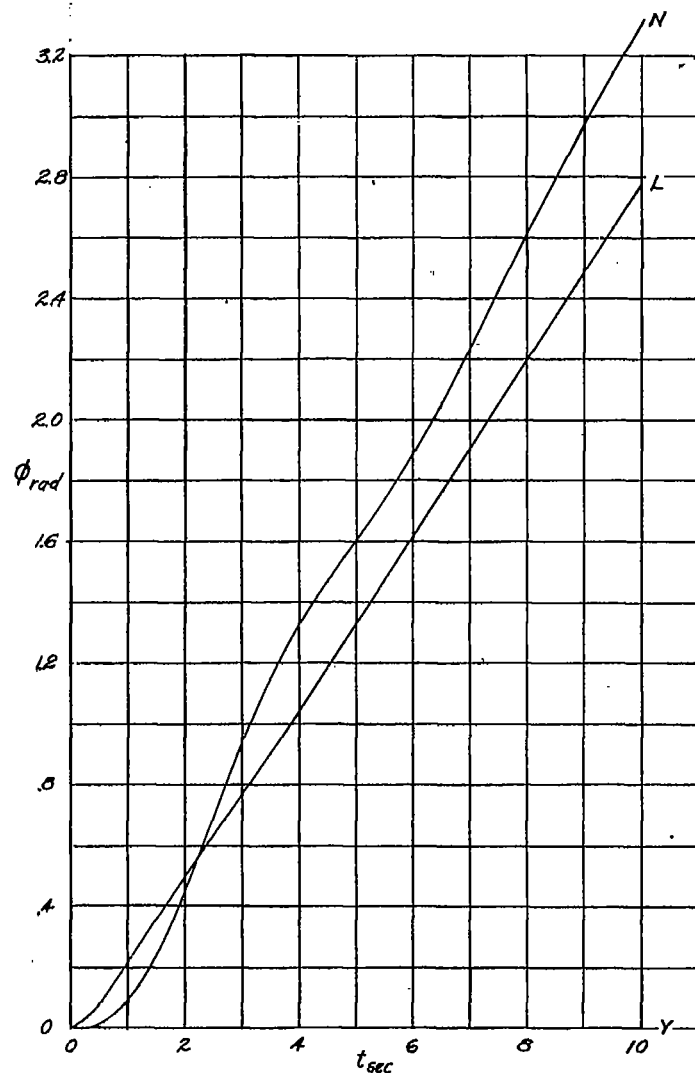


FIGURE 24.— THE VARIATION OF ANGLE OF BANK WITH TIME FOR AIRPLANE 1 SHOWING THE RELATIVE EFFECTS OF A UNIT YAWING ACCELERATION, ROLLING ACCELERATION, AND SIDE ACCELERATION.

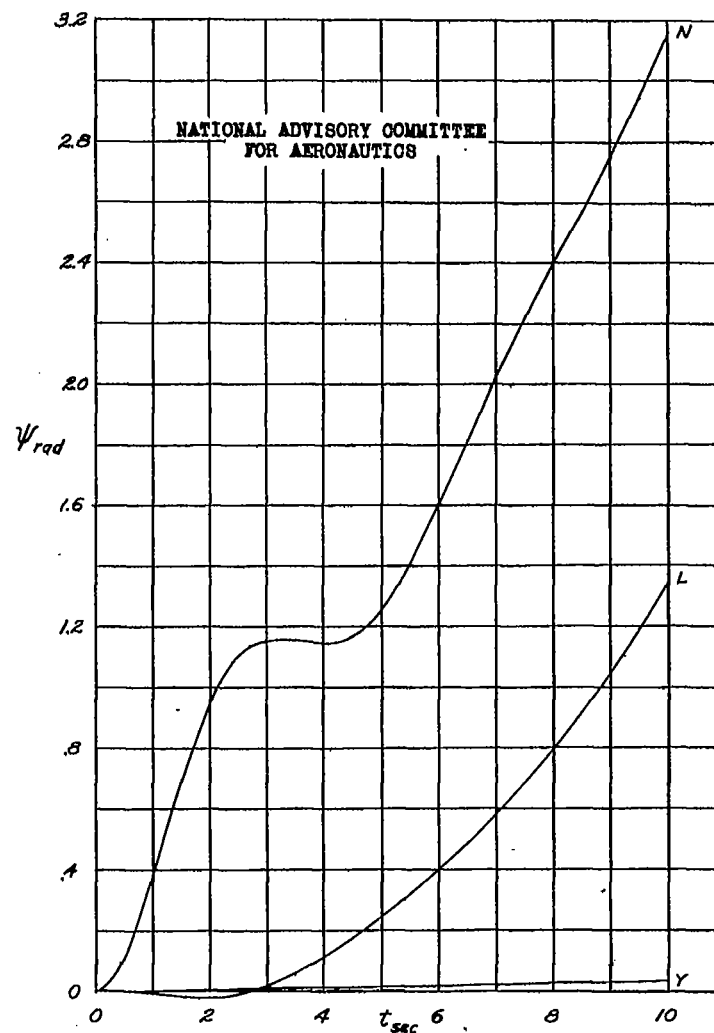


FIGURE 25.— THE VARIATION OF ANGLE OF YAW WITH TIME FOR AIRPLANE 1 SHOWING THE RELATIVE EFFECTS OF A UNIT YAWING ACCELERATION, ROLLING ACCELERATION, AND SIDE ACCELERATION.

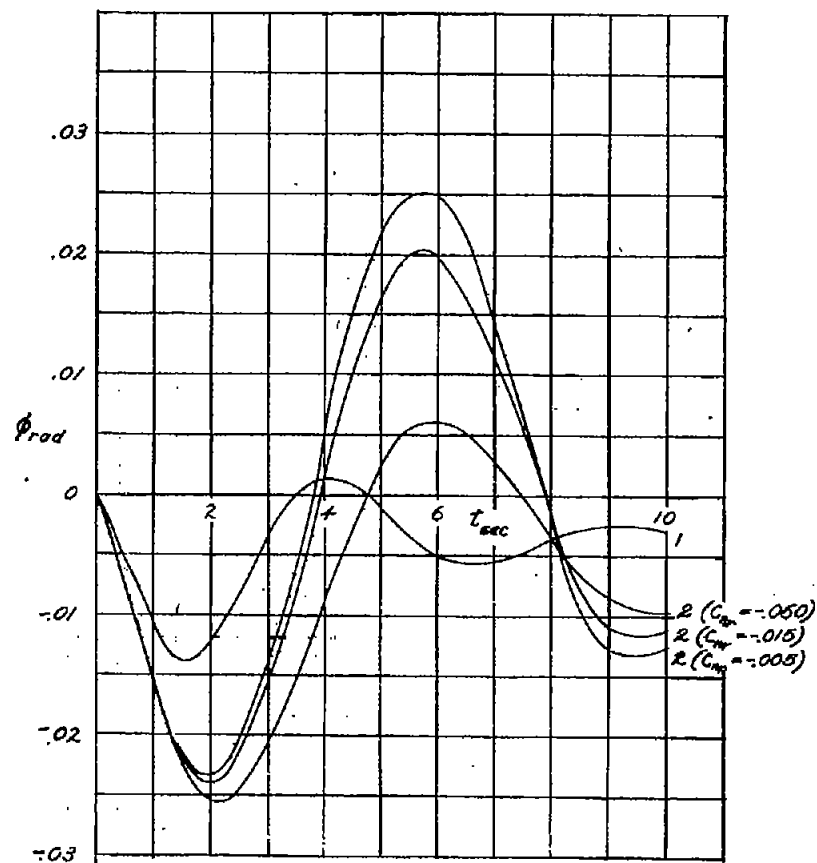


FIGURE 26. — THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE EFFECT OF A 10 FT/SEC LATERAL GUST FROM THE RIGHT (POSITIVE GUST) FOR AIRPLANES 1 AND 2.

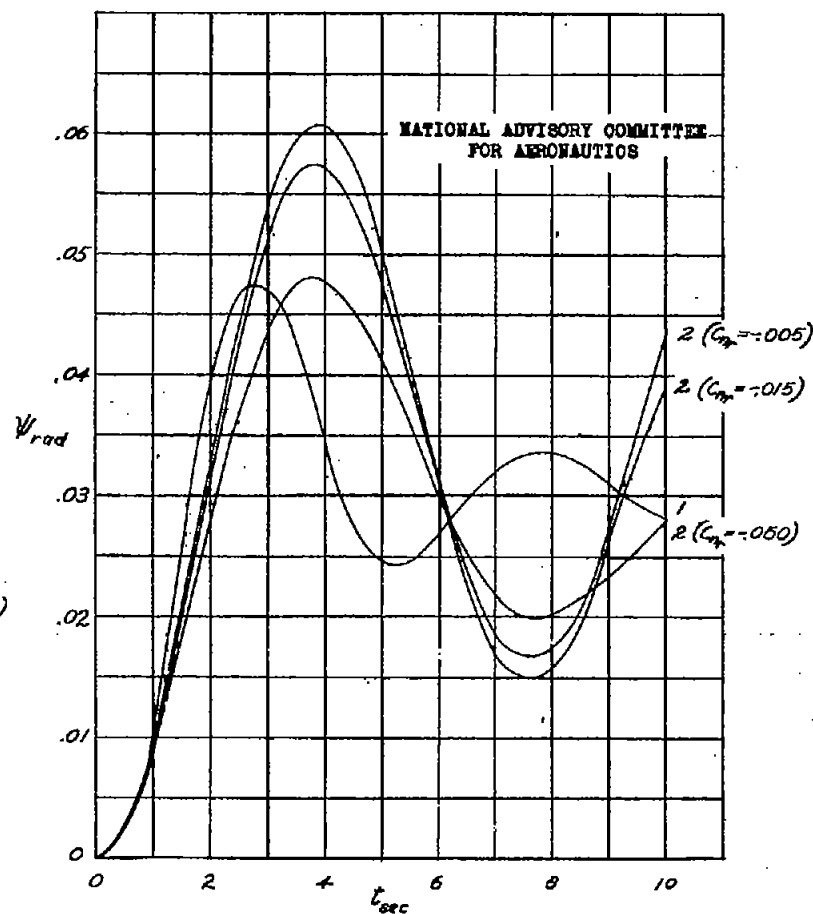


FIGURE 27. — THE VARIATION OF THE ANGLE YAW WITH TIME DUE TO THE EFFECT OF A 10 FT/SEC LATERAL GUST FROM THE RIGHT (POSITIVE GUST) FOR AIRPLANES 1 AND 2.

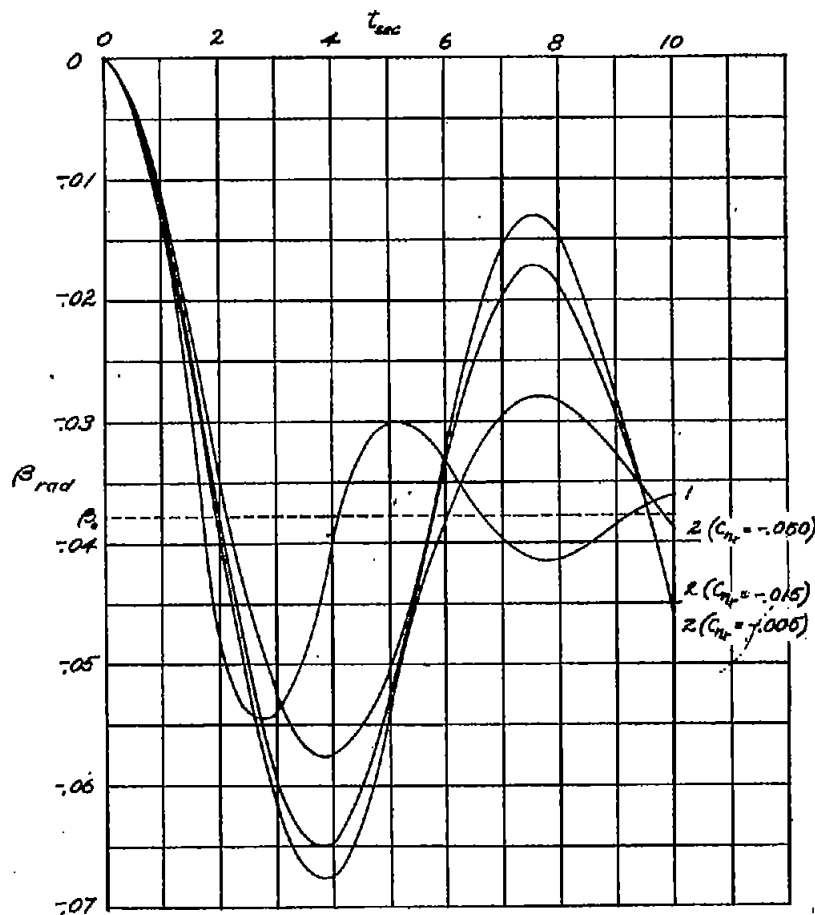


FIGURE 28.- THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE EFFECT OF A 10 FT/SEC LATERAL GUST FROM THE RIGHT (POSITIVE GUST) FOR AIRPLANES 1 AND 2.

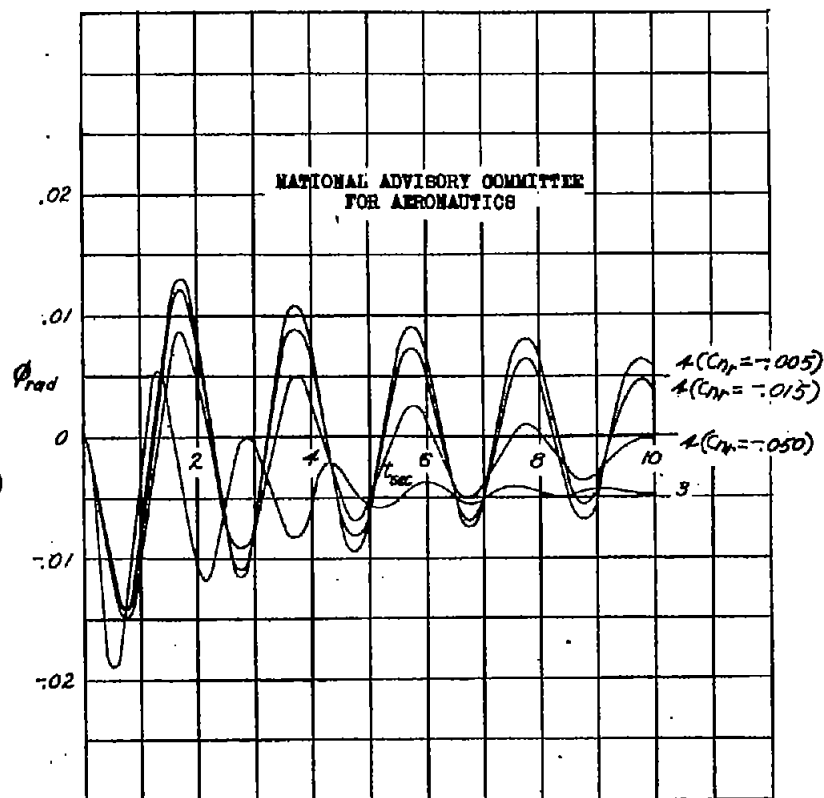


FIGURE 29.- THE VARIATION OF THE ANGLE OF BANK WITH TIME DUE TO THE EFFECT OF A 10 FT/SEC LATERAL GUST FROM THE RIGHT (POSITIVE GUST) FOR AIRPLANES 3 AND 4.

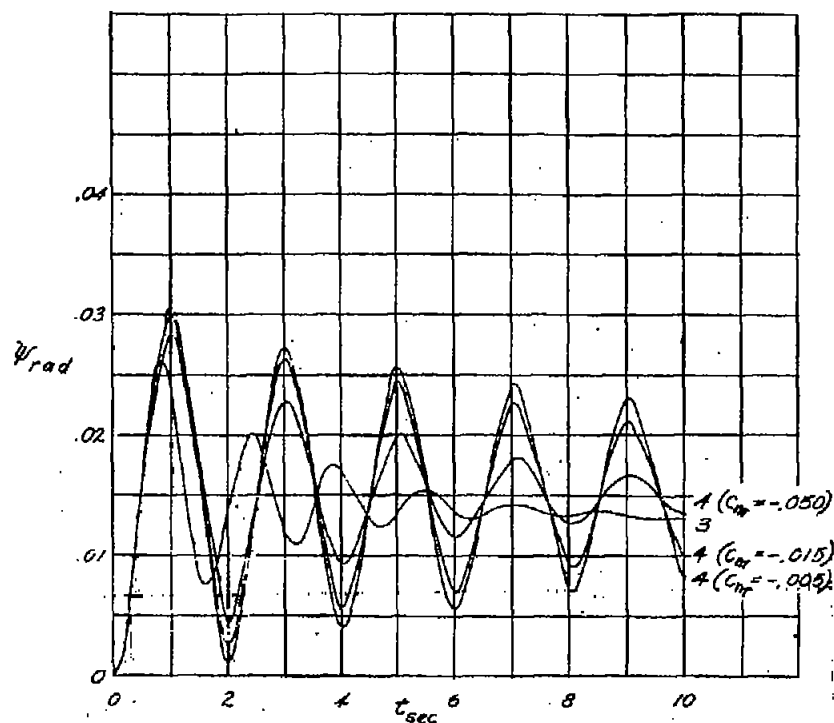


FIGURE 30.—THE VARIATION OF THE ANGLE OF YAW WITH TIME DUE TO THE EFFECT OF A 10 FT/SEC LATERAL GUST FROM THE RIGHT (POSITIVE GUST) FOR AIRPLANES 3 AND 4.

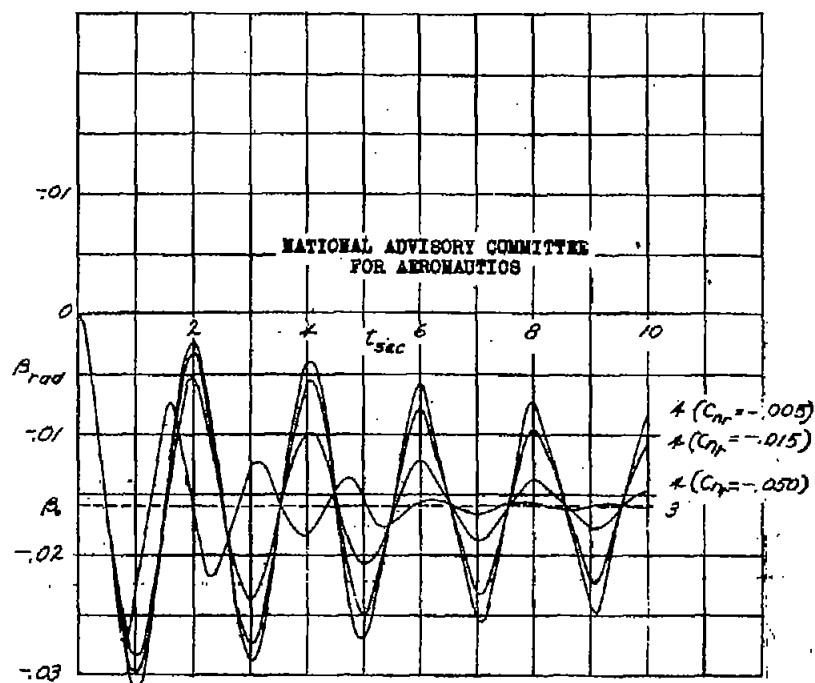


FIGURE 31.—THE VARIATION OF THE ANGLE OF SIDESLIP WITH TIME DUE TO THE EFFECT OF A 10 FT/SEC LATERAL GUST FROM THE RIGHT (POSITIVE GUST) FOR AIRPLANES 3 AND 4.

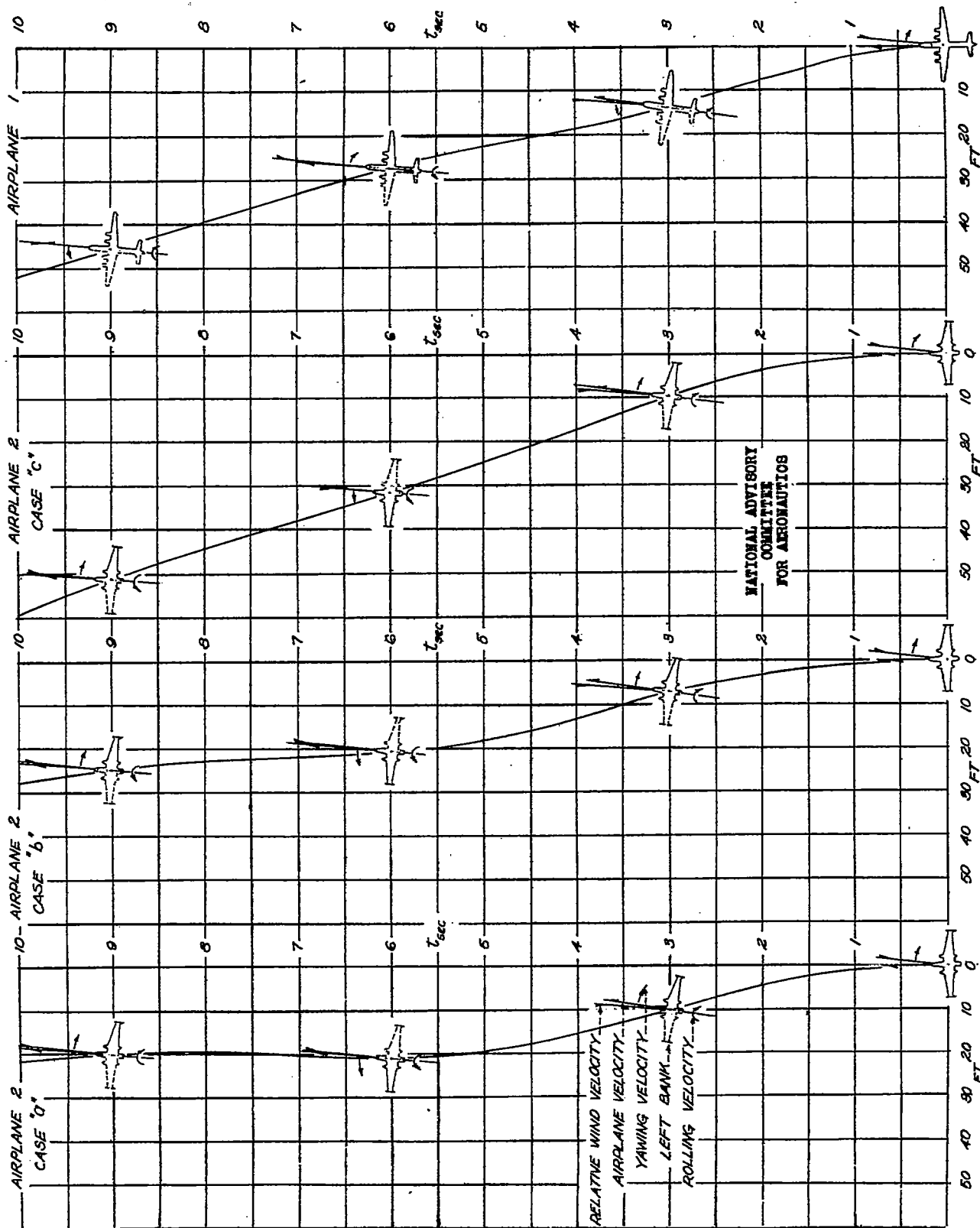


FIGURE 32, - THE PATHS OF AIRPLANES 1 AND 2 WITH RESPECT TO THE GROUND AFTER ENTRANCE INTO A 30 FT/SEC SHARP-EDGED LATERAL GUST.  $U_0 = 264.5$  FT/SEC.

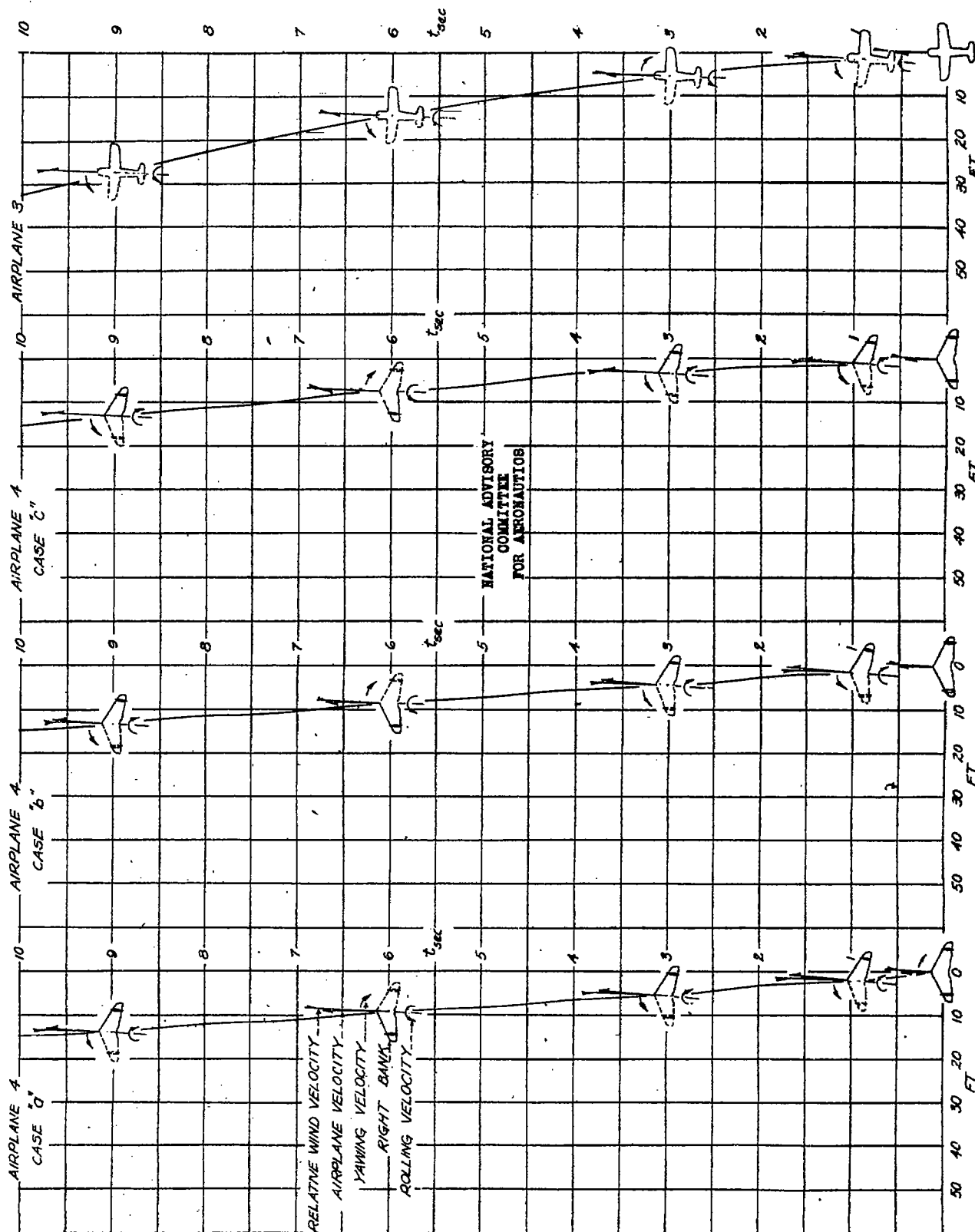


FIGURE 33. — THE PATHS OF AIRPLANES 3 AND 4 WITH RESPECT TO THE GROUND AFTER ENTRANCE INTO A 30 FT/SEC SHARP-EDGED LATERAL GUST  
 $U_0 = 626$  FT/SEC.

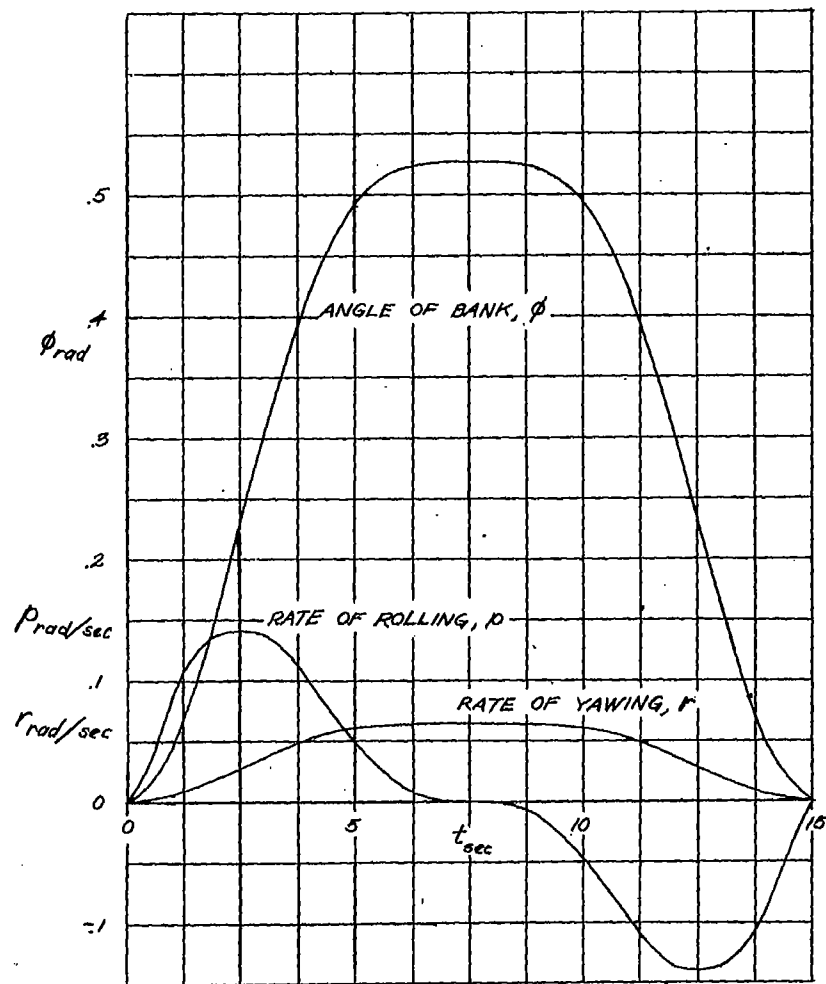


FIGURE 34. - THE VARIATION, WITH TIME, OF THE ANGLE BANK SPECIFIED, THE RATE OF ROLLING AND THE RATE OF YAWING FOR THE 15-SECOND COORDINATED TURN MANEUVER.  $U_0 = 264.5$  FT/SEC.

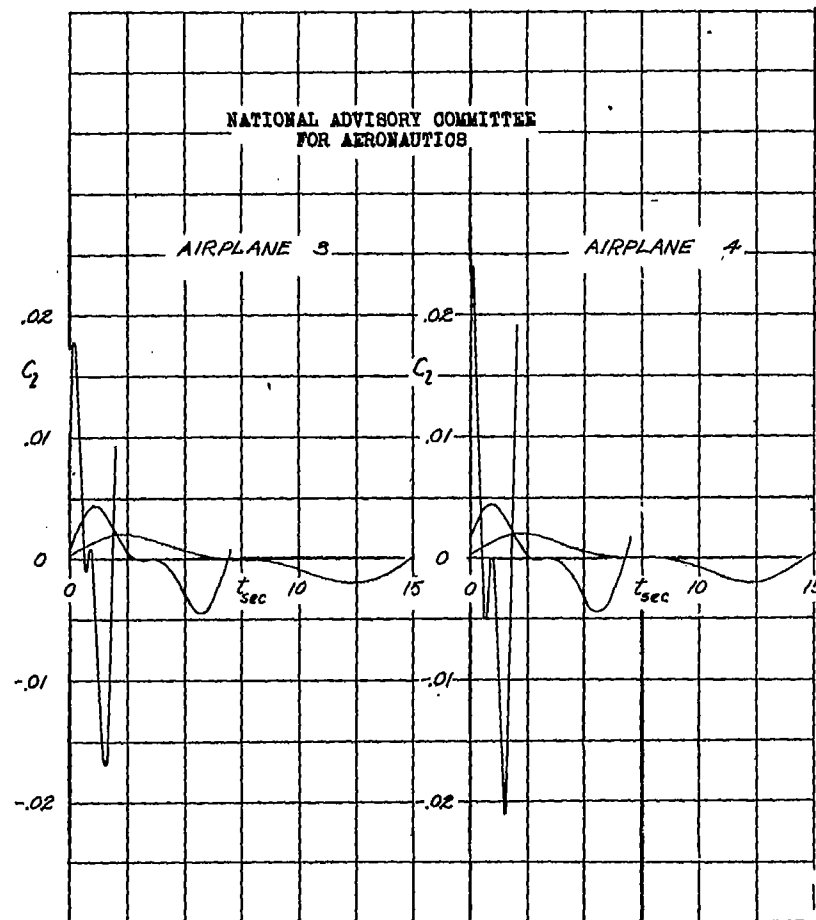


FIGURE 35. - THE VARIATION OF ROLLING-MOMENT COEFFICIENT WITH TIME REQUIRED TO COMPLETE A ZERO SIDESLIP,  $30^\circ$  BANKED TURN IN VARIOUS PERIODS OF TIME. AIRPLANES 3 AND 4.

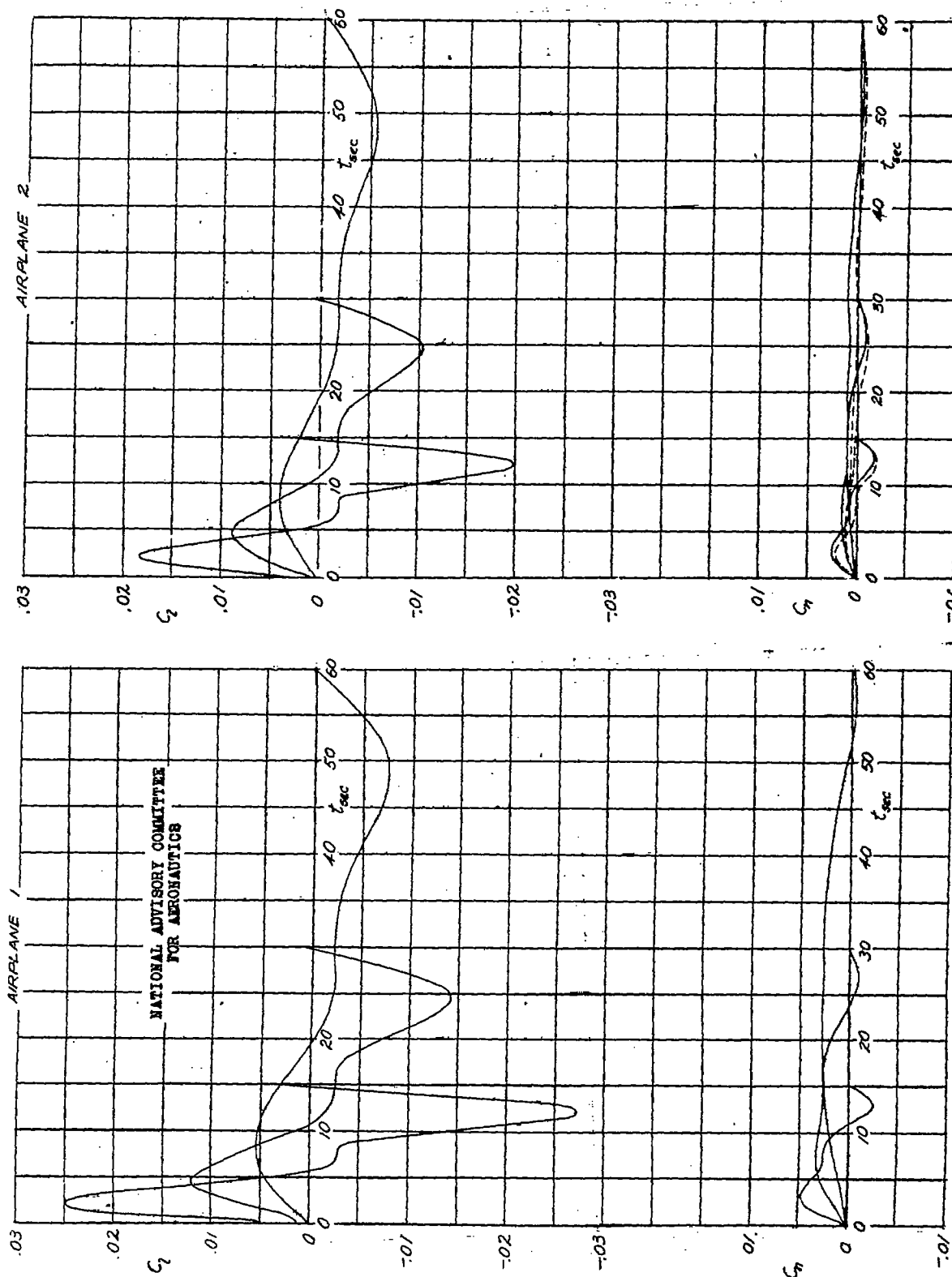
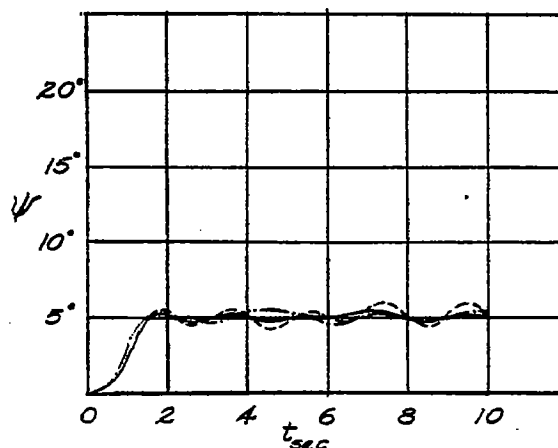
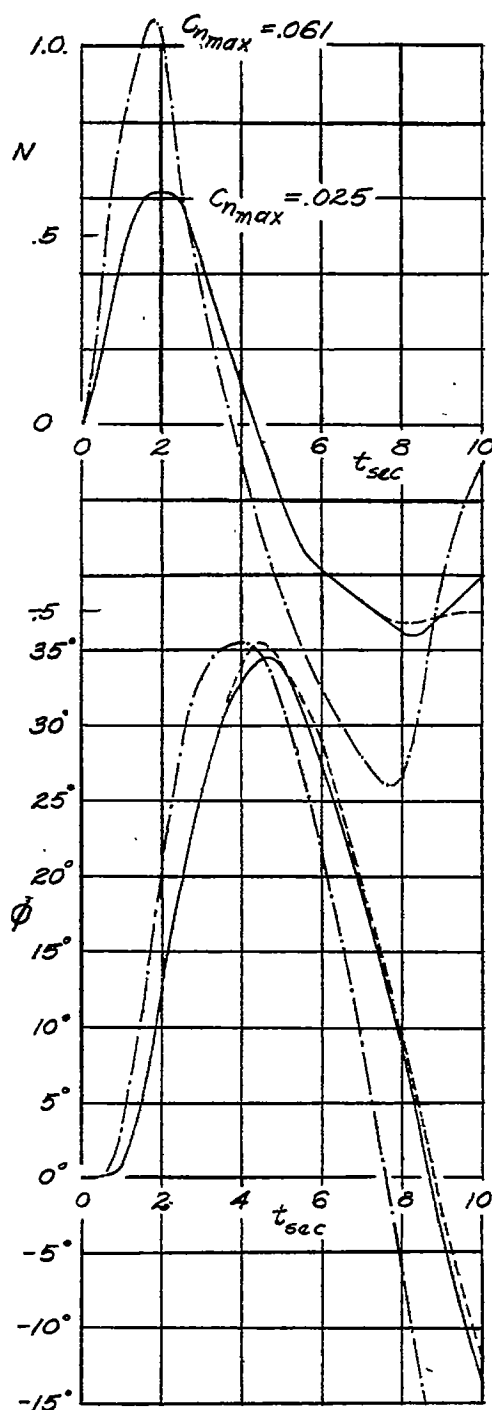


FIGURE 36. - THE VARIATION OF ROLLING-MOMENT COEFFICIENT AND YAWING MOMENT COEFFICIENT WITH TIME REQUIRED TO COMPLETE A ZERO SIDESLIP, 30° BANKED TURN IN VARIOUS PERIODS OF TIME. AIRPLANES 1 AND 2.



NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

--- AIRPLANE 3  
--- AIRPLANE 4 CASE Q  
— AIRPLANE 4 CASE C

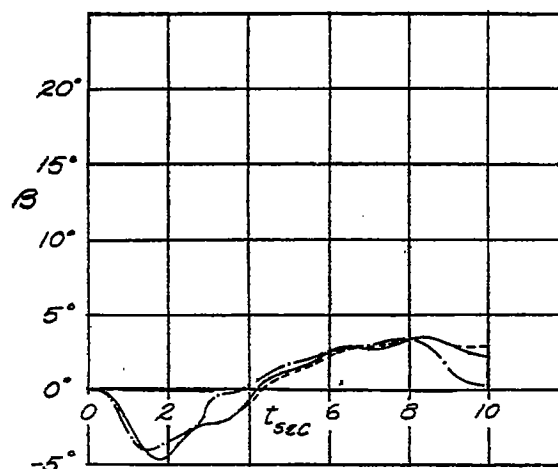
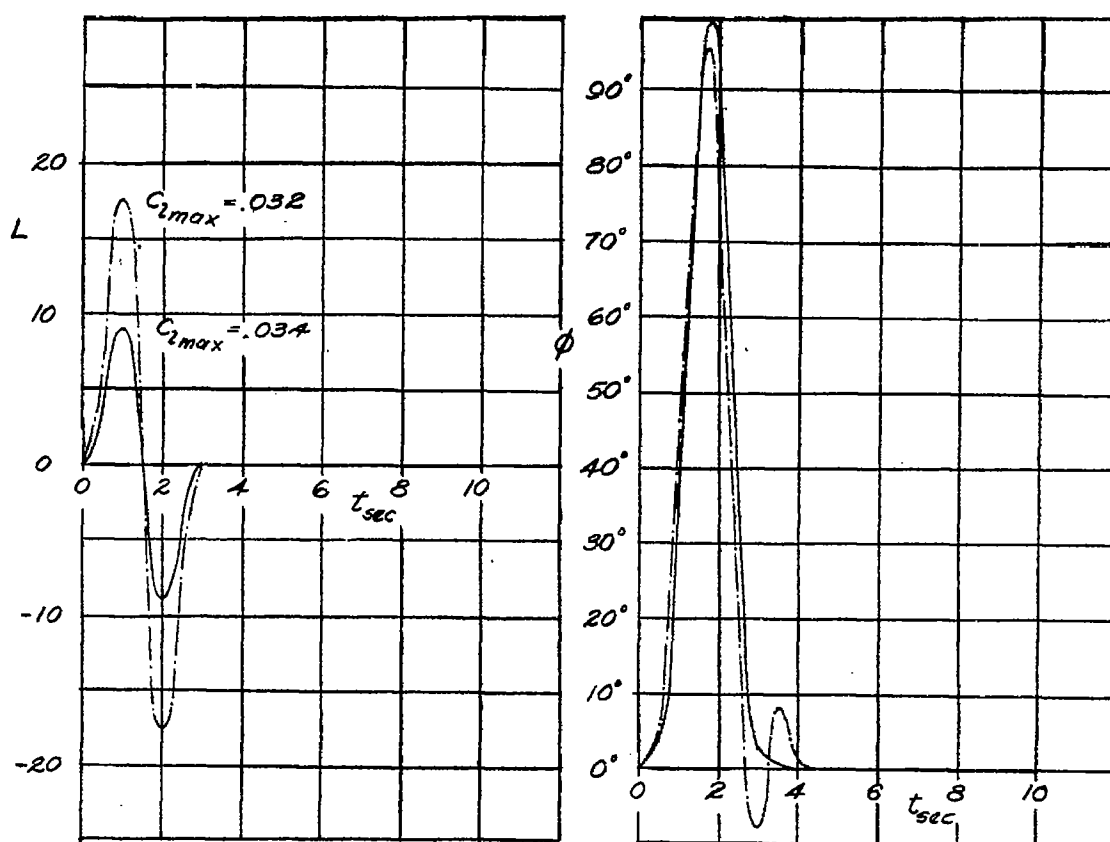


FIGURE 37.— THE VARIATION, WITH TIME, OF THE YAWING ACCELERATION REQUIRED TO ENFORCE AND HOLD A 5° CHANGE IN HEADING, AND OF THE RESULTING ANGLES OF YAW, BANK, AND SIDE-SLIP. AIRPLANES 3 AND 4 (CASE Q AND C).



NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

— AIRPLANE 3  
— AIRPLANE 4 CASE C

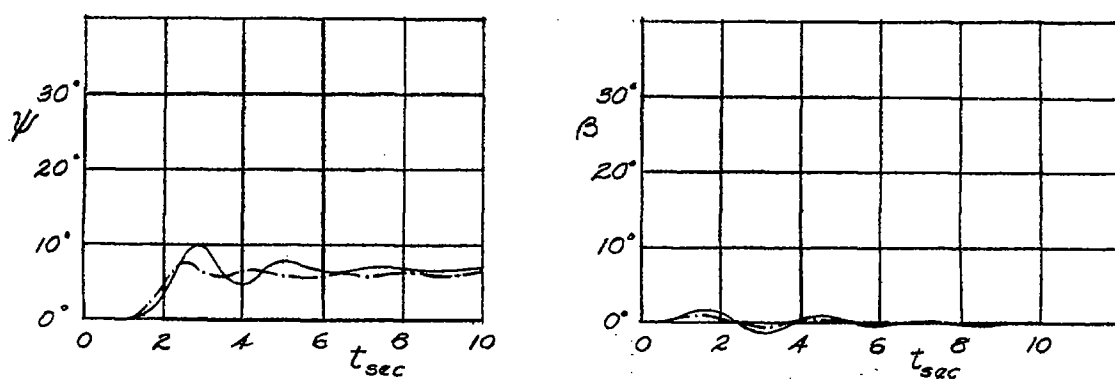
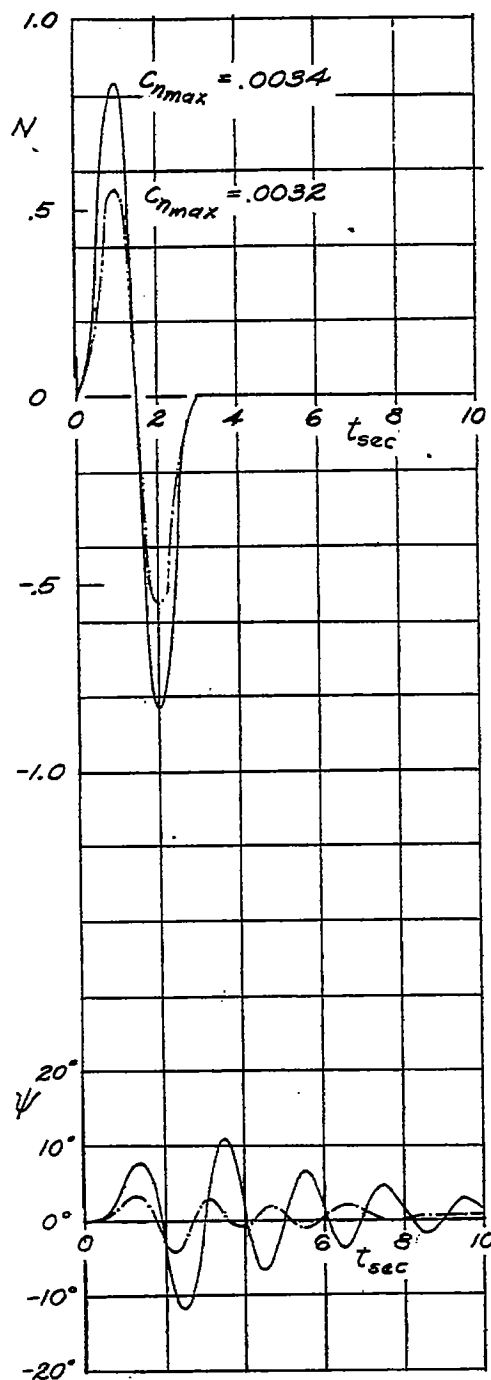
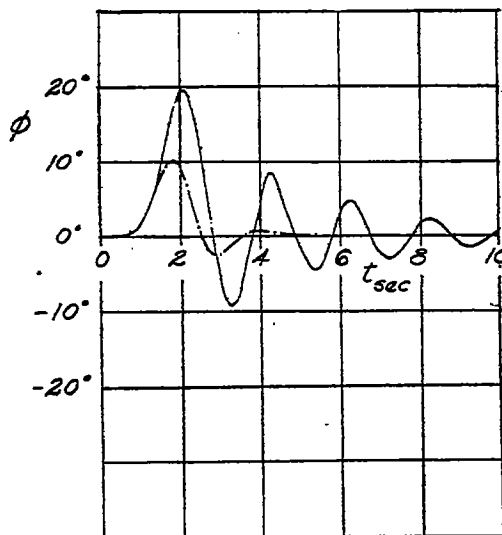


FIGURE 38.- THE VARIATION, WITH TIME, OF THE ROLLING ACCELERATION REQUIRED TO REACH APPROXIMATELY  $90^\circ$  BANK AND RETURN TO  $0^\circ$  BANK AND OF THE RESULTING ANGLES OF BANK, YAW, AND SIDESLIP. AIRPLANES 3 AND 4 (CASE C)

NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

— AIRPLANE 3  
— AIRPLANE 4 CASE "C"

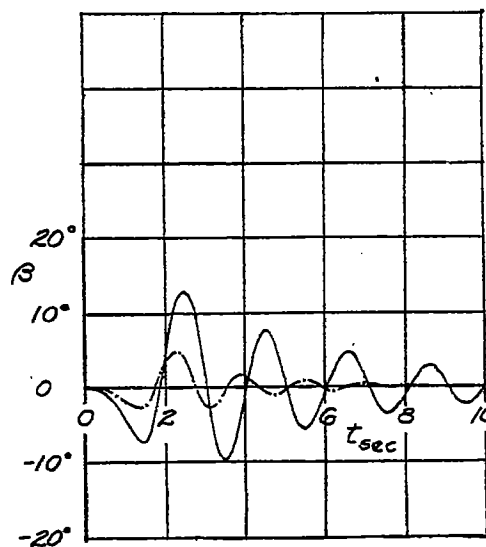
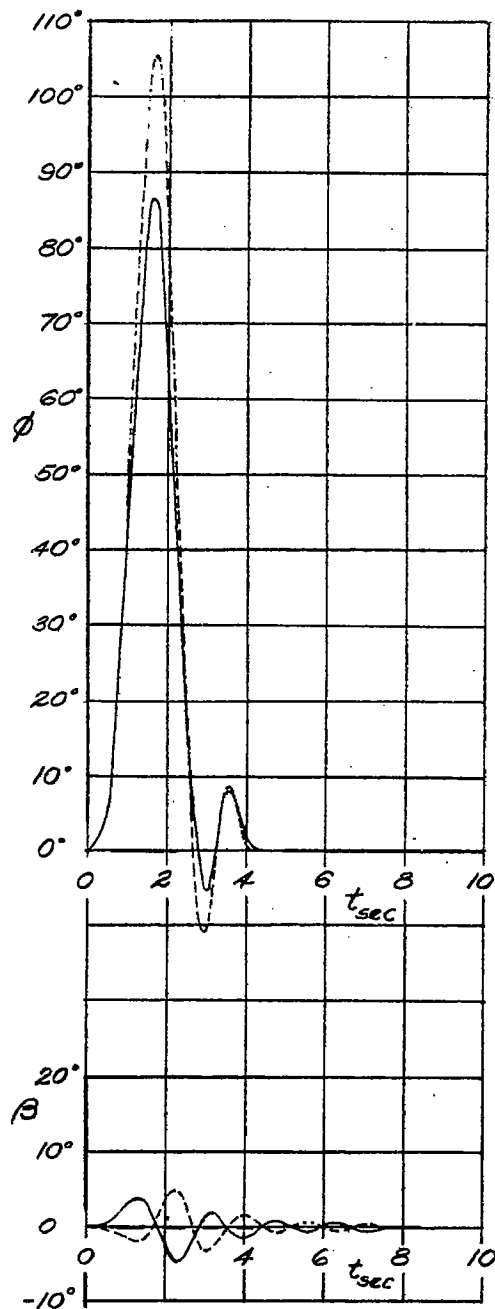


FIGURE 39.—THE VARIATION, WITH TIME, OF THE YAWING ACCELERATION DUE TO FAVORABLE AILERON YAW ( $C_{\eta} = 0.1 C_L$ ) AND OF THE RESULTING ANGLES OF BANK, YAW, AND SIDESLIP. AIRPLANES 3 AND 4 (CASE C).



NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

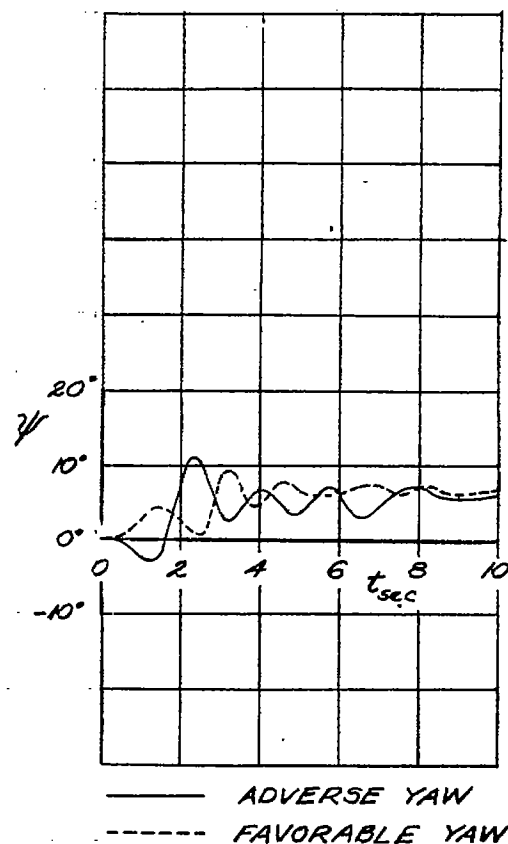


FIGURE 40.—THE EFFECT OF YAW DUE TO AILERONS ON THE VARIATION, WITH TIME, OF THE ANGLES OF BANK, YAW AND SIDESLIP DEVELOPED DURING A ROLL TO APPROXIMATELY 90° AND RETURN OF AIRPLANE 3.

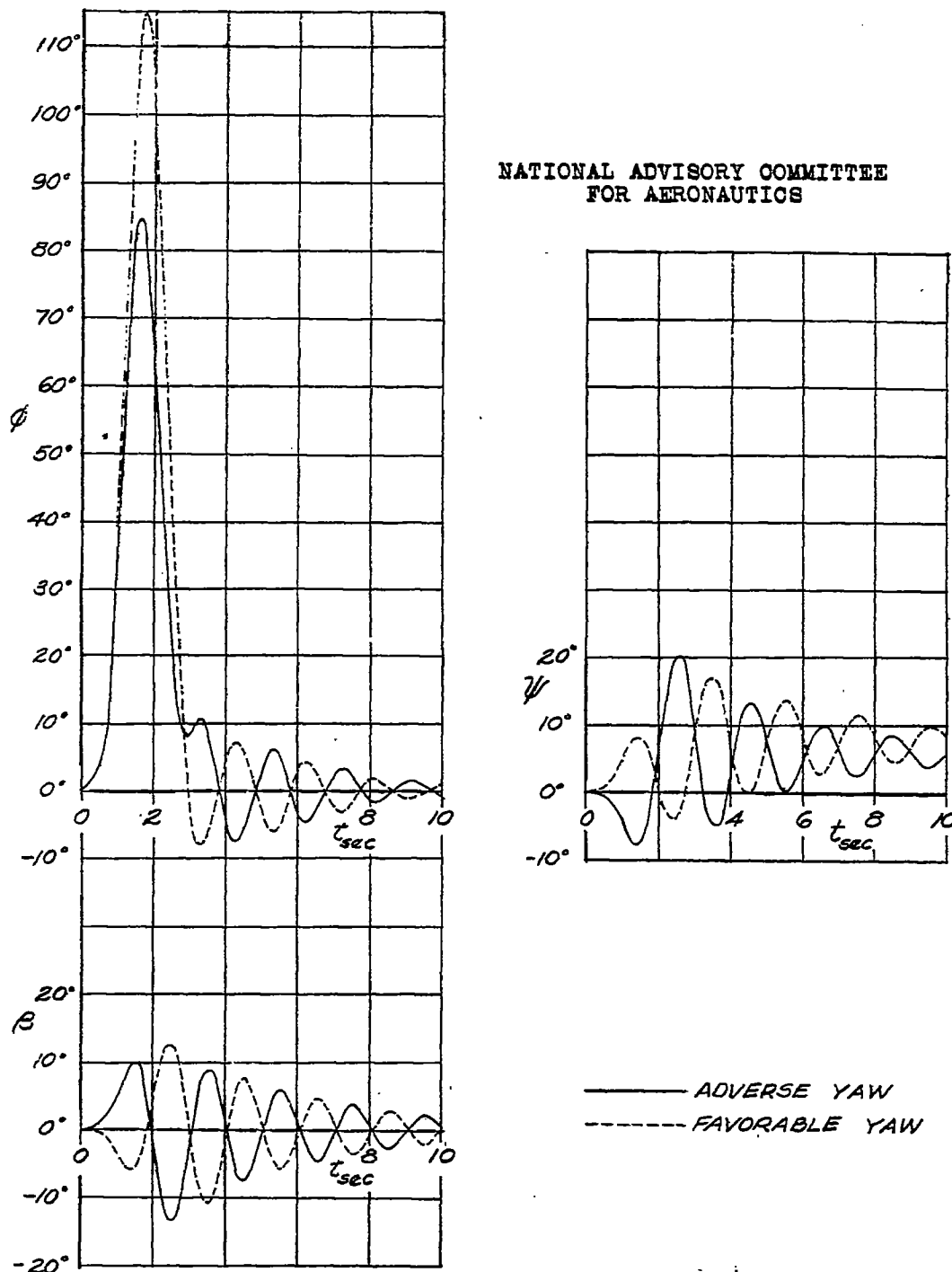


FIGURE 41.-THE EFFECT OF YAW DUE TO AILERONS ON THE VARIATION, WITH TIME, OF THE ANGLES OF BANK, YAW AND SIDESLIP DEVELOPED DURING A ROLL TO APPROXIMATELY  $90^\circ$  BANK AND RETURN OF AIRPLANE 4 (CASE C).